# Overview of Prism <br> Sensitivity vs Specificity <br> Contingency Tables <br> Fisher's Exact Test 



Biostatistics Course 2023
Lecture 1
Monday, 24 July 2023
1:00pm - 3:00pm

## Overview of Prism



## L. GraphPad

## INTRODUCING

Prism8
The preferred analysis and graphing solution purpose-built for scientific research. Join the world's leading scientists and discover how you can use Prism to save time, make more appropriate analysis choices, and elegantly graph and present your scientific research.

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Since 1989, GraphPad Software has been dedicated to creating software exclusively for the international scientific community.

Created by scientists for scientists, our intuitive programs provide researchers worldwide with the tools they need to simplify data analysis, statistics and graphing.


Dr. Harvey Motulsky
Founder

## Committed to quality

GraphPad is committed to providing an exceptional level of customer service. Over the years, we have listened to our customers and used their suggestions to make our programs easier, more powerful, and more versatile.

## Knowledge and experience

GraphPad Software was founded by Dr. Harvey Motulsky, who at the time was on the faculty of the Dept. of Pharmacology at the University of California San Diego. In addition to doing research in receptor pharmacology, Dr. Motulsky also taught biostatistics to medical and graduate students and wrote the text Intuitive Biostatistics. Dr. Motulsky's pharmacology and statistics background make him uniquely qualified to provide researchers with the software features and support they require.


Motulsky, 2017 Intuitive Biostatistics 4th Edition
"Intuitive Biostatistics is both an introduction and review of statistics.Compared to other books, it has:

- Breadth rather than depth. It is a guidebook, not a cookbook.
- Words rather than math. It has few equations.
- Explanations rather than recipes. This book presents few details of statistical methods and only a few tables required to complete the calculations....
I wrote Intuitive Biostatistics for three audiences:
- Medical (and other) professionals who want to understand the statistical portions of journals they read. These readers don't need to analyze any data, but need to understand analyses published by others. I've tried to explain the big picture, without getting bogged down in too many details.
* Undergraduate and graduate students, post-docs and researchers who will analyze data. This book explains general principles of data analysis, but it won't teach you how to do statistical calculations or how to use any particular statistical program. It makes a great companion to the more traditional statistics texts and to the documentation of statistical software.
- Scientists who consult with statisticians. Statistics often seems like a foreign language, and this text can serve as a phrase book to bridge the gap between scientists and statisticians. Sprinkled throughout the book are "Lingo" sections that explain statistical terminology, and point out when statistics gives ordinary words very specialized meanings (the source of much confusion)."


Bremmer \& Doerge, 2009


Bremmer \& Doerge, 2015

- Present basic statistical equations (without derivation).
- Best read linearly, not just as references (despite the titles).
- A good refresher for those who have had some statistics training.
- Does not provide as much intuition or practical guidance as Motulsky.
- For novices I recommend reading this after Motusky's book.
- 2009 book provides recipes to use in Microsoft Excel (best to avoid doing this)
- 2015 book provides recipes to use in R. R is much more powerful than GrapPad Prism, but it's also much easier to mess up statistical calculations in R.
- I recommend using Prism unless you have confidence in your understanding of statistical equations.

Contingency table: sensitivity vs. specificity

## Porphyria is a class of diseases caused by impaired heme synthesis



We focus on Acute Intermediate Porphyria, which is caused by loss-of-function mutations in porphobilinogen deaminase and leads to a build-up of porphobilinogen.

porphobilinogen


Bissell et al., 2017, NEJM
"The typical patient with an attack of acute intermittent porphyria is a previously healthy young woman who has had several days of severe fatigue and an inability to concentrate, followed by progressively worsening abdominal pain, nausea, vomiting, and subtle neurologic signs."

There is a screening test for AIP, based on the measurement of reduced levels of porphobilinogen deaminase (PBGD) activity in urine or serum.


## Question:

If you test positive for AIP in this screening test, what is the probability that you actually have AIP?

Sensitivity is the probability of testing positive given that the subject has the disease.

For the AIP test:

$$
\text { Sensitivity }=p\left(\text { test }^{+} \mid \text {disease }^{+}\right)=82 \%
$$



Specificity is the probability of a negative test given that the subject does not have the disease.

For the AIP test:

Specificity $=p\left(\right.$ test $^{-} \mid$disease $\left.^{-}\right)=96.3 \%$


Prevalence is the fraction of individuals in a population who have a disease.

Understanding the results of a medical screening test requires also knowing the prevalence of a disease

> For AIP:
> Prevalence $=p\left(\right.$ disease $\left.^{+}\right)=0.01 \%$



Contingency table showing the expected results of the AIP test on 1,000,000 random individuals



Positive predictive value (PPV):


Even if you test positive, the probability of you having AIP is still very, very low.



If a subject's sibling has AIP, there is a $50 \%$ chance that they do too.

$$
\text { prevalence }=p\left(\text { disease }^{+}\right)=50 \%
$$



## Consider the expected outcome in 1,000,000 individuals with affected siblings




Positive predictive value (PPV):

Just knowing that you sibling has AIP increases the
PPV of the test enormously.

## The influence of population is a key reason that doctors distinguish between screening tests and diagnostic tests

|  | Screening tests | Diagnostic tests |
| :---: | :---: | :---: |
| Purpose | To detect potential disease indicators | To establish presence/absence of disease |
| Target population | Large numbers of asymptomatic, but potentially at risk individuals | Symptomatic individuals to establish diagnosis, or asymptomatic individuals with a positive screening test |
| Test method | Simple, acceptable to patients and staff | maybe invasive, expensive but justifiable as necessary to establish diagnosis |
| Positive <br> result <br> threshold | Generally chosen towards high sensitivity not to miss potential disease implies many FPs! | Chosen towards high specificity (true negatives). More weight given to accuracy and precision than to patient acceptability |
| Positive result | Essentially indicates suspicion of disease (often used in combination with other risk factors) that warrants confirmation | Result provides a definite diagnosis |
| Cost | Cheap, benefits should justify the costs since large numbers of people will need to be screened to identify a small number of potential cases | Higher costs associated with diagnostic test maybe justified to establish diagnosis. |

https://www.healthknowledge.org.uk/public-health-textbook/disease-causation-diagnostic/ 2c-diagnosis-screening/screening-diagnostic-case-finding

The relationship between prevalence, sensitivity, specificity, and PPV is clarified by considering "odds"

| posterior odds | likelihood ratio | prior odds |  |
| :---: | :---: | :---: | :---: |
| $\frac{p\left(\text { disease }^{+} \mid \text {test }^{+}\right)}{p\left(\text { disease }^{-} \mid \text {test }^{+}\right)}$ | $\frac{p\left(\text { test }^{+} \mid \text {disease }^{+}\right)}{p\left(\text { test }^{+} \mid \text {disease }^{-}\right)}$ | $\left.\frac{)}{-}\right) \times \frac{p\left(\text { disease }^{+}\right)}{p\left(\text { disease }^{-}\right)}$ |  |
| 11 | II | II |  |
| PPV | sensitivity | prevalence |  |
| 1-PPV | 1-specificity | 1-prevalence |  |
| (what you care about) | (property of test) | (property of population) |  |
| $\left[0.0022=\frac{0.22 \%}{99.78 \%}\right]$ | $=\left[22.2=\frac{82.0 \%}{3.7 \%}\right]$ | $\times\left[10^{-4}=\frac{0.01 \%}{99.99 \%}\right]$ | random individual |
|  | II |  |  |
| $\left[22.2=\frac{95.7 \%}{4.3 \%}\right]$ | $=\left[22.2=\frac{82.0 \%}{3.7 \%}\right]$ | $\times \quad\left[1=\frac{50 \%}{50 \%}\right]$ | sibling of affected individual |

The base rate fallacy describes the human tendency to discount prior information

## posterior $=$ likelihood $\times \quad$ prior odds $\quad$ ratio $\times$ odds

base rate fallacy: If presented with related base rate information (i.e. generic, general information) and specific information (information pertaining only to a certain case), the mind tends to ignore the former and focus on the latter.

The "population" an individual comes from, and thus prior odds, are greatly affected by many hard-to-quantify factors

- Has the individual had any relevant symptoms?
- Does the individual have a relevant family history?
- What is the individual's ethnicity (ancestry)?
-What is the individual's sex?
- Has the individual been tested before? How?

Prior odds aren't a property of an individual per se, but rather one's state of knowledge about that individual.

Prior odds (and thus posterior odds) quantify subjective uncertainty.

## Statistics is divided into two schools: Frequentist and Bayesian.

Frequentist statistics avoids calculations involving prior odds.
It therefore yields results that are prone to misinterpretation due the base rate fallacy.

However, frequentist statistics is used heavily in biological research, so you have to learn it anyway.

Frequentist statistics is still useful and informative if you know what to watch out for.

Bayesian statistics explicitly accounts for prior odds.
It therefore requires prior information that is often hard to quantify.
Bayesian statistics is central to modern machine learning and more advanced areas of quantitative biology.

Experimental researchers in biology tend not use Bayesian statistics, so in this specific course won't discuss it much.


Contingency tables: Each row defines a treatment or exposure, each column defines an outcome, and each value is an exact count of objects or events

| Table format <br> Contingency |  | A | B |
| :---: | :---: | :---: | :---: |
|  |  | Cases | Control |
| $\mathbf{1}$ |  | Smoked |  |
| Y |  |  |  |
| $\mathbf{2}$ | Never smoked |  |  |



Data table:


Fishers exact test of retrospective data (smoking and cancer)


Sensitivity and specificity (HIV)
-square test for trend


| - $0^{\circ}$ |  | D lec1_aip_test.pzfx - Edited |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q Search |  |  | Table format: Contingency |  | Outcome A | Outcome B | Outcome C |
| Data Tables <br> Sensitivity and specificity (AIP) |  |  |  |  | AIP disease + | AIP disease - | Title |
|  |  |  |  | $\checkmark$ | Y | Y | Y |
| $\oplus$ New Data Table... <br> Info <br> (i) Project info 1 <br> $\oplus$ New Info... <br> - Results <br> New Analysis... <br> (4) M Graph... <br> $\checkmark$ Layouts <br> $\oplus$ New Layout... |  | 》 | 1 | PBGD test + | 82 | 36996 |  |
|  |  |  | 2 | PBGD test - | 18 | 962904 |  |
|  |  | " | 3 | Title |  |  |  |
|  |  |  | 4 | Title |  |  |  |
|  |  | » | 5 | Title |  |  |  |
|  |  |  | 6 | Title |  |  |  |
|  |  |  | 7 | Title |  |  |  |
|  |  | » | 8 | Title |  |  |  |
|  |  |  | 9 | Title |  |  |  |
|  |  | 》 | 10 | Title |  |  |  |
| \# Sensitivity and specificity (AIP) <br> $\triangle$ Sensitivity and specificity (AIP) |  | ) | 11 | Title |  |  |  |
|  |  |  | 12 | Title |  |  |  |
| $\boxed{W}$ Sensitivity and specificity (AIP) |  |  | 13 | Title |  |  |  |
|  |  |  | 14 | Title |  |  |  |
|  |  |  | 15 | Title |  |  |  |
|  |  |  | 16 | Title |  |  |  |
|  |  |  | 17 | Title |  |  |  |
|  |  |  | 18 | Title |  |  |  |
|  |  |  | 19 | Title |  |  |  |
|  |  |  | 20 | Tithe |  |  |  |
| $\square$ - $\downarrow$ - | [ ${ }^{\text {a }}$ |  |  |  | Sensitivity and specificity (AI $\square$ $0^{2}$ Row 6, $\Theta$ ${ }^{+}$ |  |  |

## Data to analyze

Table: Sensitivity and specificity (AIP)

Type of analysis

Which analysis?

- Transform, Normalize...

Transform
Transform concentrations (X)
Normalize
Prune rows
Remove baseline and column math
Transpose X and Y
Fraction of Total

- XY analyses
- Column analyses
- Grouped analyses

V Contingency table analyses
Chi-stuare (and Fisher's exact) test
Row r ns with SD or SEM
Fract Total

- Surviva an alyses
- Parts of whole analyses
- Multiple variable analyses
- Nested analyses
- Generate curve
- Simulate data
- Recently used



## Effect sizes to report

Relative Risk
Used for prospective and experimental studiesDifference between proportions (attributable risk) and NNT
Used for prospective and experimental studiesOdds ratio
Used for retrospective case-control studies
Sensitivity, specificity and predictive values
ed for diagnostic tests
Me, Add to compute the $P$ valueFisher's exact testYates' continuity corrected chi-square testChi-square testChi-square test for trend

Looking for the $z$ test to compare proportions? Choose the chi-square test (with or without the Yates' correction). The chi-square and $z$ tests are equivalent.


| B lec1＿aip＿test．pzfx－Edited |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q－Search | Contingency |  |  |  |  |  |
| $\checkmark$ Data Tables |  |  |  |  |  |  |
| \＃Sensitivity and specificity（AIP） |  |  |  |  |  |  |
| $\oplus$ New Data Table．．． <br> Info <br> （i）Project info 1 <br> $\oplus$ New Info．．． <br> Results | 1 | Table Analyzed | Sensitivity and specificity（AIP） |  |  |  |
|  | ＞ 2 |  |  |  |  |  |
|  | 3 | P value and statistical significance |  |  |  |  |
|  | 》 4 | Test | Fisher＇s exact test |  |  |  |
| $\square$ Contingency of Sensitivity and specificity（AIP） | 5 | P value | ＜0．0001 |  |  |  |
| New Analysis．．． <br> Graphs Sensitivity and specificity（AIP） New Graph．．． <br> Layouts New Layout．．． | 6 | P value summary | ＊＊＊＊ |  |  |  |
|  | 》 7 | One－or two－sided | Two－sided |  |  |  |
|  | 8 | Statistically significant（ $P<0.05$ ）？ | Yes |  |  |  |
|  | 》 9 |  |  |  |  |  |
|  | 16 | Effect size | Value | 95\％CI |  |  |
|  | 1. | Sensitivity | 0.8200 | 0.7333 to 0.8830 |  |  |
| Family | \＃ 1 | Specificity | 0.9630 | 0.9626 to 0.9634 |  |  |
| \＃Sensitivity and specificity（AIP） | －1 | Positive Predictive Value | 0.002212 | 0.001782 to 0.002744 |  |  |
| Contingency | 12 | Negative Predictive Value | 1.000 | 1.000 to 1.000 |  |  |
|  | 15 ？ | Likelihood Ratio | 22.16 |  |  |  |
|  | 16 memer |  |  |  |  |  |
|  | 17 | Methods used to compute Cls |  |  |  |  |
|  | 18 | Sensitivity，specificity，etc． | Wilson－Brown |  |  |  |
|  | 19 |  |  |  |  |  |
|  | 20 | Data analyzed | AIP disease＋ | AIP disease－ | Total |  |
|  | 21 | PBGD test＋ | 82 | 36996 | 37078 |  |
|  | 22 | PBGD test－ | 18 | 962904 | 962922 |  |
|  | 23 | Total | 100 | 999900 | 1000000 |  |
|  | 24 |  |  |  |  |  |
|  | 囲（i） | $\underline{\sim}$ | ity and specificity（A） | Row 1，Column A | Q－ |  |

Contingency table: prospective study

Does taking aspirin daily affect one's chance of myocardial infarction (MI)


Null hypothesis:
Aspirin usage has no effect on Ml risk
Alternative hypothesis:
Aspirin increases or decreases MI risk.

## Statistical test:

Fisher's exact test

## Statistical test: Fisher's exact test



## Mathematical formalization:

Is there a statistical dependence between the row an observation falls in and the column that observation falls in?

## Null hypothesis:

There is no statistical dependence: $p($ row,column $)=p(r o w) \times p($ column $)$

## Alternative hypothesis:

There is a statistical dependence: $p($ row,column $)=p($ row $) \times p$ (column)



Contingency tables: Each row defines a treatment or exposure, each column defines an outcome, and each value is an exact count of objects or events

| Table format <br> Contingency |  | A | B |
| :---: | :--- | :---: | :---: |
|  |  | Cases | Control |
| $\mathbf{1}$ | Smoked |  | Y |
| $\mathbf{2}$ | Never smoked |  |  |



Data table:



## Data to analyze

Table: Prospective (aspirin and MI)

Type of analysis
Which analysis?
$\boldsymbol{\nabla}$ Transform, Normalize...
Transform
Transform concentrations (X)
Normalize
Prune rows
Remove baseline and column math
Transpose $X$ and $Y$
Fraction of Total

- XY analyses
- Column analyses
- Grouped analyses

V Contingency table analyses
Chi-square (and Fisher's exact) test
Row mea vith SD or SEM
Fraction
Survival analy es

- Parts of whole analyses
- Multiple variable analyses
- Nested analyses
- Generate curve
- Simulate data
- Recently used



## Effect sizes to report

- Relative Risk
ed for prospective and experimental studies
efference between proportions (attributable risk) and NNT
d for prospective and experimental studies
O Jds ratio
Used for retrospective case-control studiesSensitivity, specificity and predictive values
Used for diagnostic tests


## Method to compute the $\mathbf{P}$ value

Fisher's exact testYates' continuity corrected chi-square testChi-square testChi-square test for trendLooking for the $z$ test to compare proportions? Choose the chi-square test (with or without the Yates' correction). The chi-square and $z$ tests are equivalent.



- P value: $<0.0001\left(^{(* * *)}\right.$, is highly significant, so we reject the null hypothesis, concluding that Aspirin affects MI risk.
- Relative risk: 1.8 [1.4 to 2.3] meaning that NOT taking Aspirin increases risk of MI.
- Reciprocal of relative risk: 0.55 [.43 to .70] meaning that taking Aspirin reduces risk of MI.
- Attributable risk: $0.77 \%$ [ $0.46 \%$ to $1.08 \%$ ] quantifies how much the probability of Ml decreases due to taking Asprin
- Number Needed to treat (NNT): 130 [92 to 215] quantifies how many individuals would need to take Aspirin in order for one to avoid a Ml event.

Caveats: Quantifications of risk apply only to Ml events during the observational period used in the study; they do not quantify lifetime risk which of course will be higher.

Contingency table: retrospective study


Doll \& Hill, British Med. J. (1950)

Null hypothesis:
Smoking does not affect lung cancer risk
Alternative hypothesis:
Smoking increases or decreases lung cancer risk

## Statistical test:

Fisher's exact test


Contingency tables: Each row defines a treatment or exposure, each column defines an outcome, and each value is an exact count of objects or events

| Table format <br> Contingency |  | A | B |
| :---: | :---: | :---: | :---: |
|  |  | Cases | Control |
| $\mathbf{1}$ | Smoked |  | $Y$ |
| 2 | Never smoked |  |  |

## Data table:



Contingency

Enter or import data into a new table
Start with sample data to follow a tutorial
Select data set:
R Fishers exact test of retrospective data (smoking and cancer)
ensitivity and specificity (HIV)
square test for trend

Open a File
LabArchives
Clone a Graph
Graph Portfolio
EXISTING FILE
Open a File
LabArchives
Clone a Graph
Graph Portfolio


## Data to analyze

Table: Retrospective (smoking and cancer)

Type of analysis

Which analysis?
$\boldsymbol{\nabla}$ Transform, Normalize...
Transform
Transform concentrations (X)
Normalize
Prune rows
Remove baseline and column math
Transpose $X$ and $Y$
Fraction of Total

- XY analyses
- Column analyses
- Grouped analyses

V Contingency table analyses
Chi-square (and Fisheris exact) test
Row means with SD or Fraction of Total

- Survival analyses
- Parts of whole analyses
- Multiple variable analyses
- Nested analyses
- Generate curve
- Simulate data
- Recently used

Analyze which data sets?
$\checkmark$ A:Cases (lung cancer)B:Control


## Effect sizes to report

Relative Risk
Used for prospective and experimental studiesDifference between proportions (attributable risk) and NNT
Used for prospective and experimental studies
4 Odds ratio
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sprsitivity, specificity and predictive values
Used for diagnostic tests

## Method to compute the $\mathbf{P}$ value

Fisher's exact testYates' continuity corrected chi-square testChi-square testChi-square test for trendLooking for the $z$ test to compare proportions? Choose the chi-square test (with or without the Yates' correction). The chi-square and $z$ tests are equivalent.


- P value: $<0.0001\left(^{(* * * *)}\right.$, is highly significant, so we reject the null hypothesis, concluding that smoking and cancer are associated.
- Odds ratio: 3.0 [1.8 to 4.9] meaning that smoking is associated with a nearly 3 -fold higher odds of getting cancer.
- Reciprocal of odds ratio: 0.34 [.20 to .55] NOT smoking is associated with a nearly 3 -fold decrease in the odds of getting cancer.

Caveats: These results are from a a retrospective study, so we can't conclude that smoking causes cancer, only that it is associated with cancer.

## Relative risk vs. Odds ratio

| Cancer <br> (event) |  | No Cancer <br> (no event) |  |
| :---: | :---: | :---: | :---: |
| Smoker | $a$ | $b$ | Total |
|  | $c$ | $d$ | $a+b$ |
| Total | $a+c$ | $b+d$ | $c+d$ |
|  | $c$ |  |  |

Risk is the probability of an event
Risk for smokers: $a /(a+b)$
Risk for nonsmokers: $c /(c+d)$

$$
\text { Relative risk: } \frac{a /(a+b)}{c /(c+d)}
$$

Odds is the probability of an event divided by the probability of no event
Odds for smokers: $a / b$
Odds for nonsmokers: $c / d$
Odds ratio: $\frac{a / b}{c / d}$

Odds is not affected by the relative number of events vs. no events, and is preferable when this ratio reflects the design of the study, not natural phenomena.

## Questions?

