Binomial tests
Chi square tests
P-values
Confidence Intervals
Null Hypothesis testing

Biostatistics Course 2024
Lecture 2
Tuesday, 9 July 2024
10:00am - 12:00pm

Example 1: Human Sex Ratio

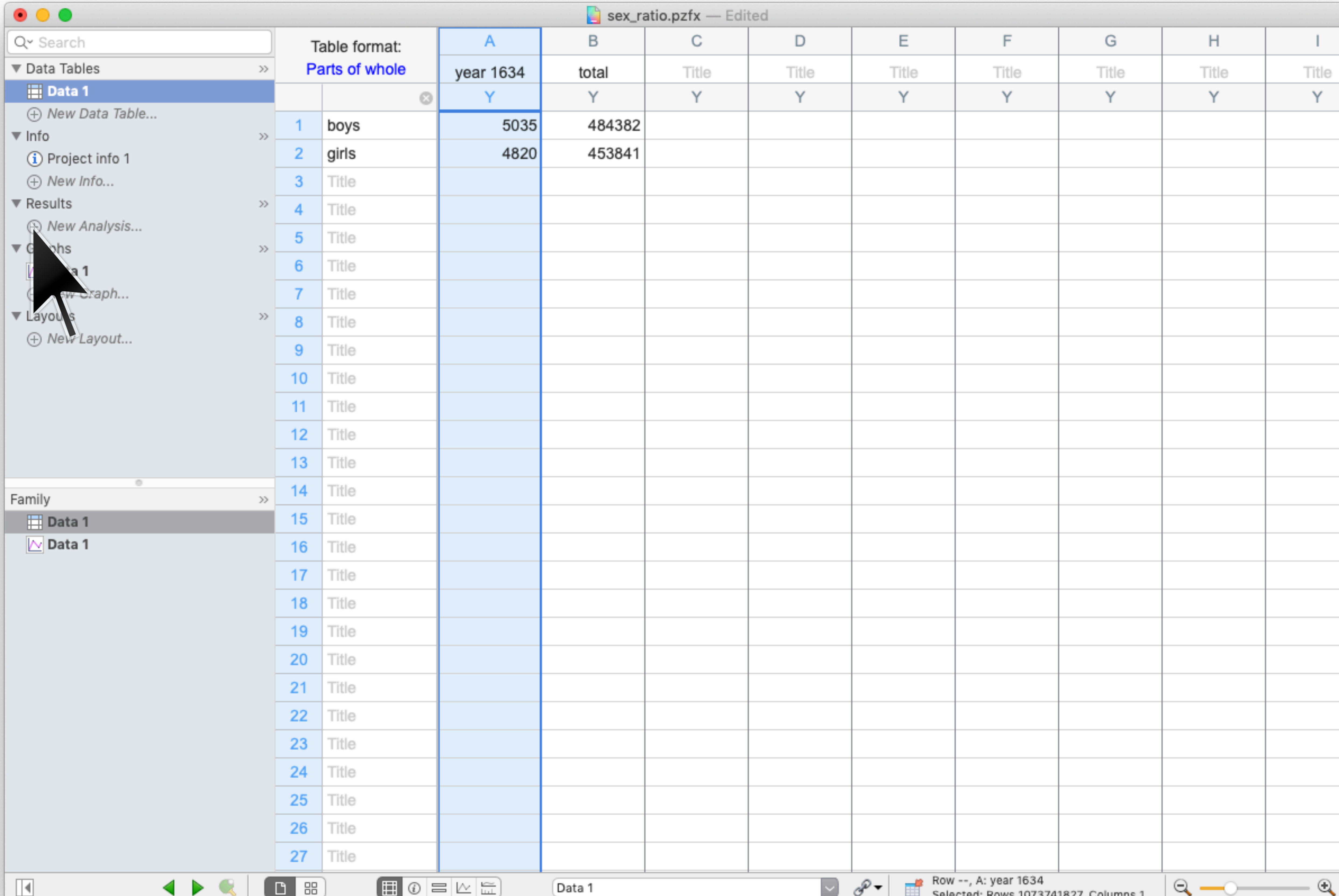
Computing sex ratio of humans is one of the oldest applications of statistics

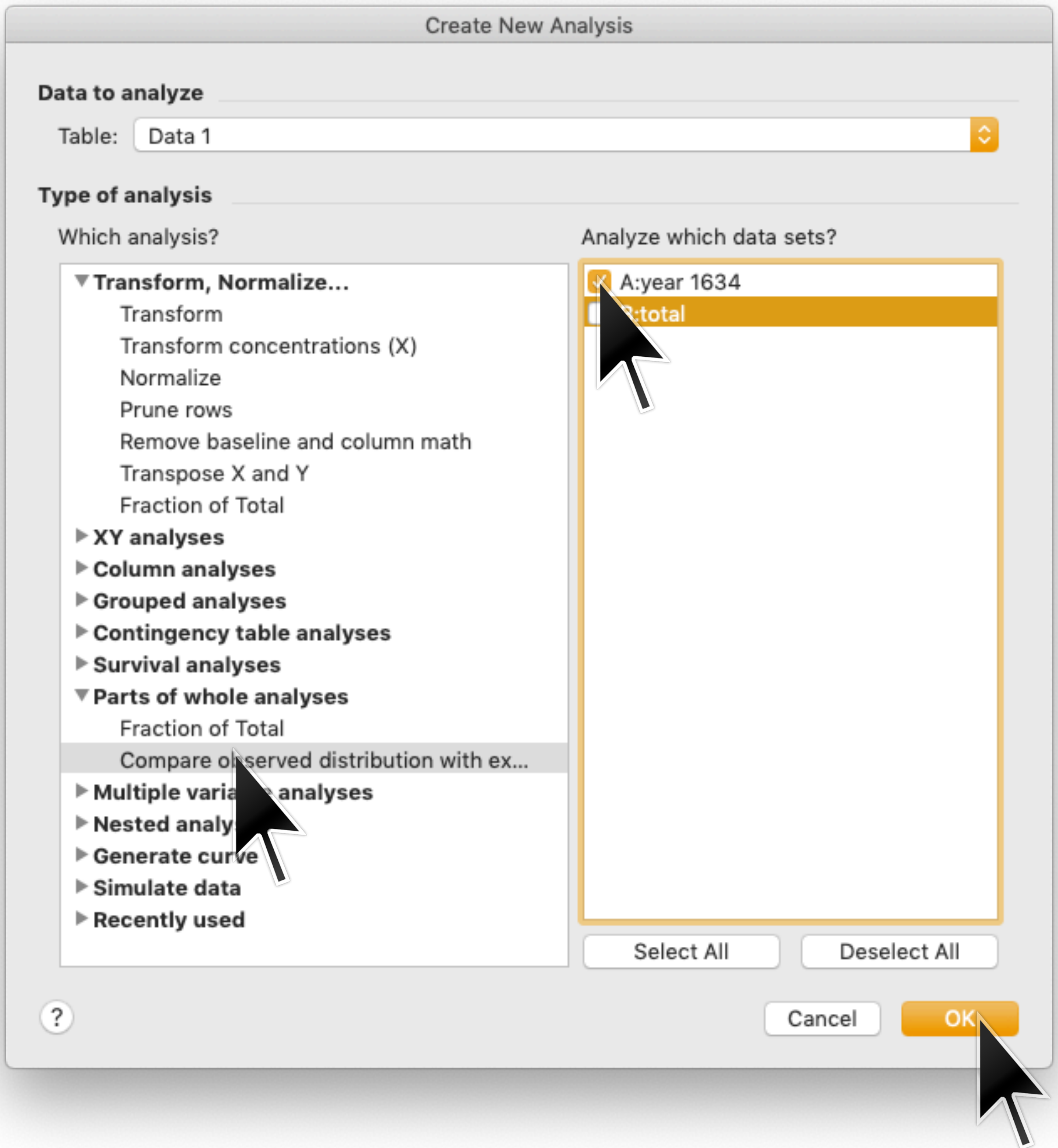
year	male	female
1629	5218	4683
1630	4858	4457
1631	4422	4102
1632	4994	4590
1633	5158	4839
1634	5035	4820
1635	5106	4928
1636	4917	4605
1637	4703	4457

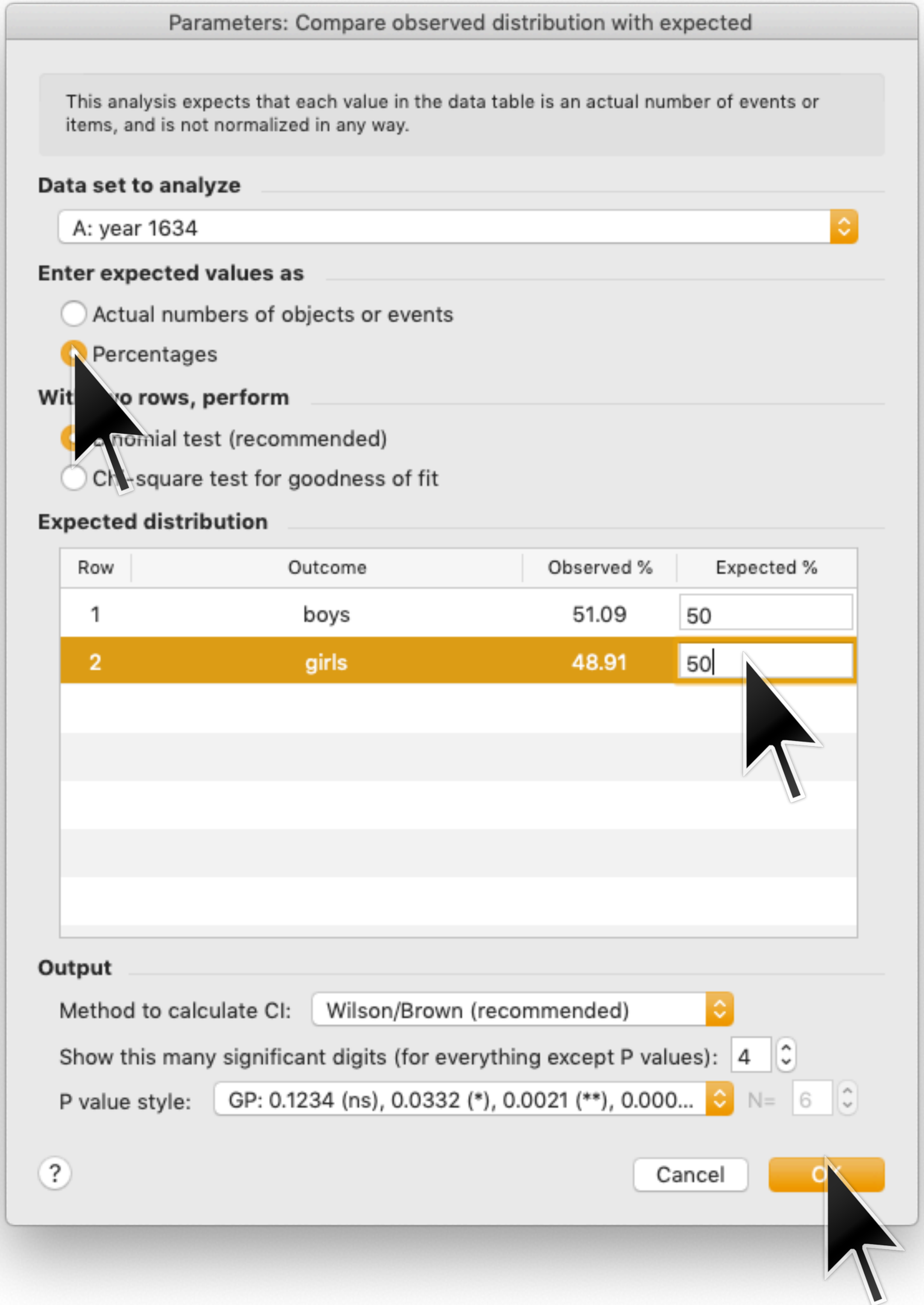
⋮

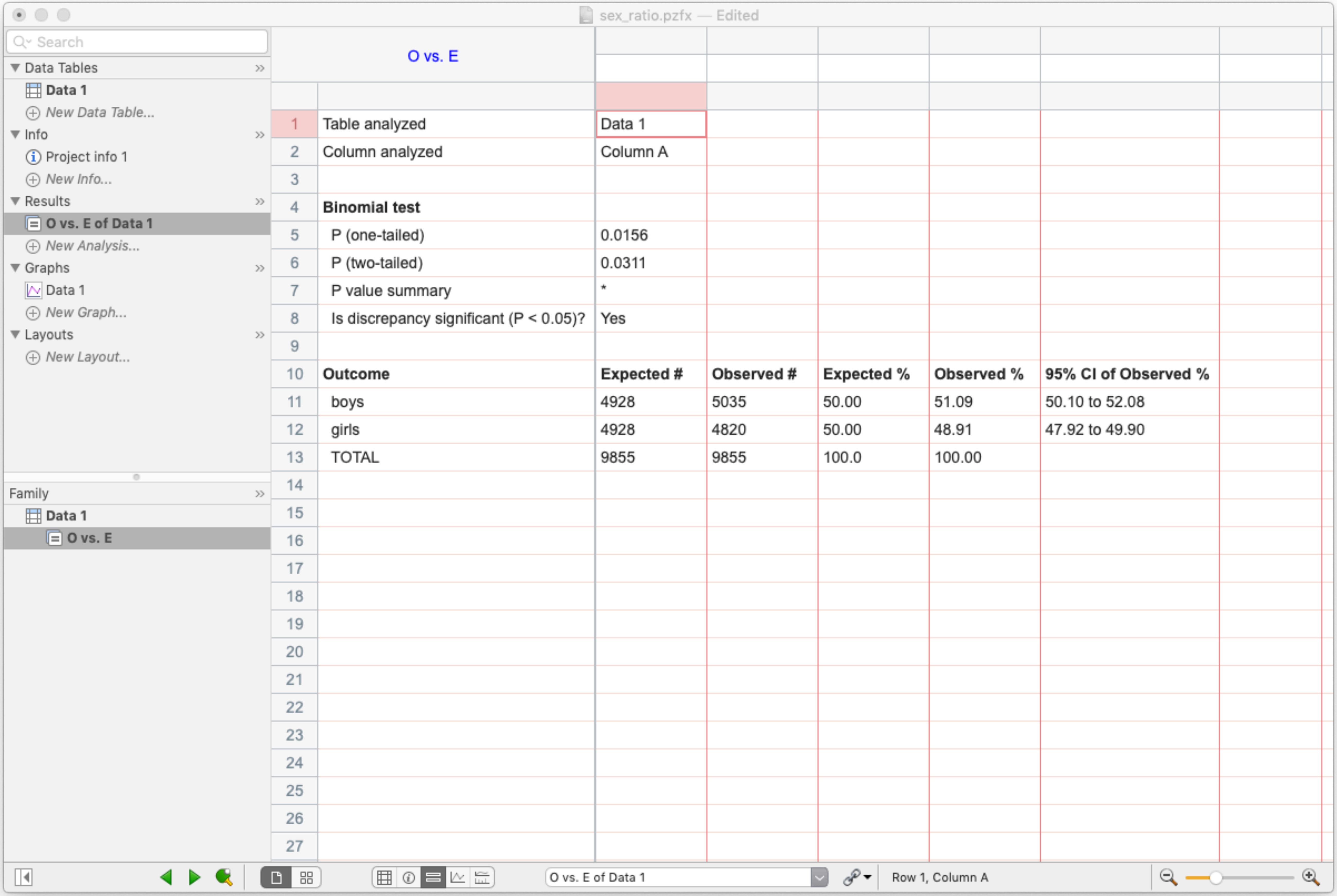
Arbuthnott J (1711). An Argument for Divine Providence, taken from the Constant Regularity observed in the Births of both Sexes.

https://www.openintro.org/stat/data/?data=arbuthnot


sex_ratio.pzfx — Edited
Search
Data Tables
Data 1
New Data Table...
Info
Project info 1
New Info...
Results
New Analysis...
Layouts
New Layout...
Family
Data 1
Data 1
Table format:
Parts of whole
A
B
C
D
E
F
G
H
I
year 1634
total
Title
Y
1 boys 5035 484382
2 girls 4820 453841
Data 1
Row --, A: year 1634
Selected: Rows 1073741827, Columns 1

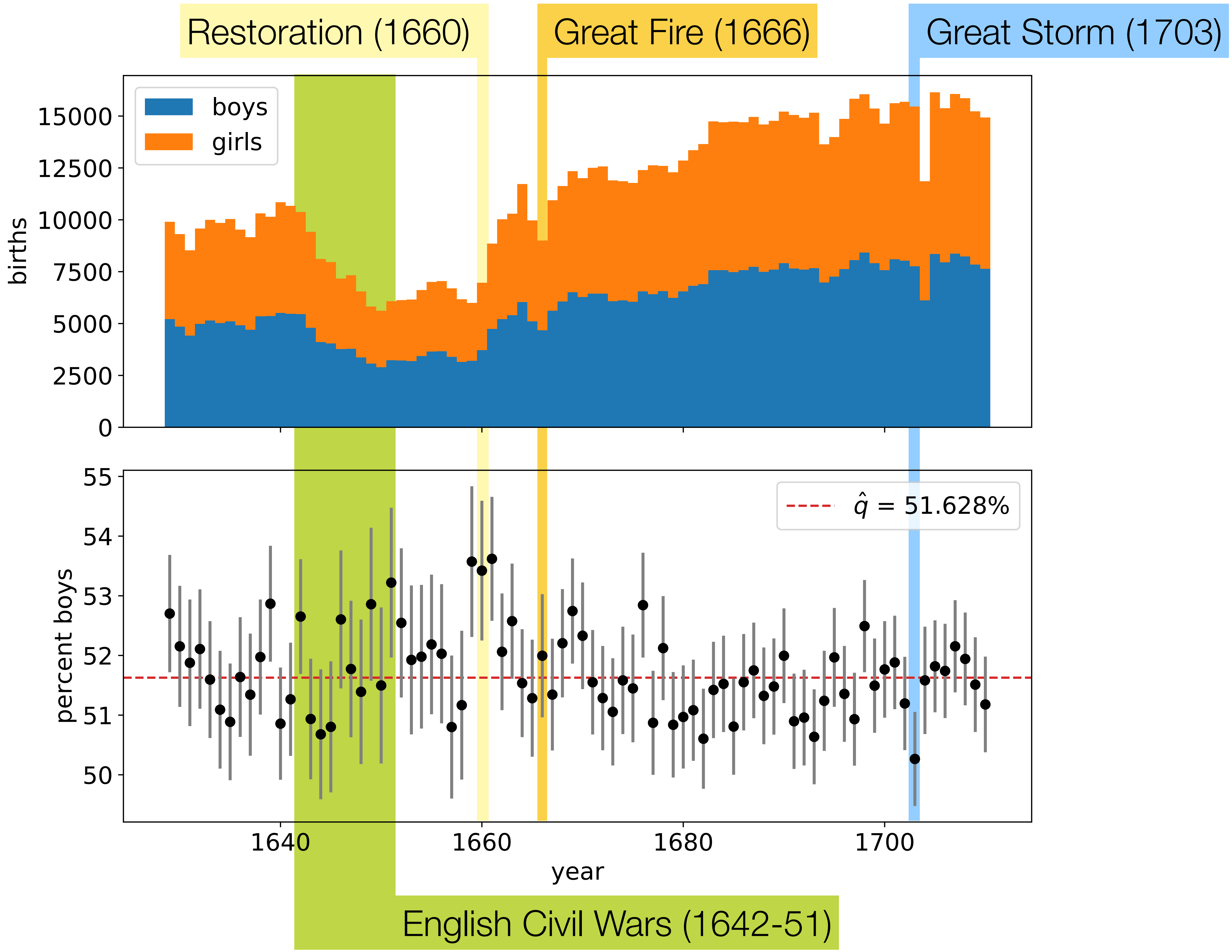

Create New Analysis
Data to analyze
Table: Data 1
Type of analysis
Which analysis?
Analyze which data sets?
Transform, Normalize…
Transform
Transform concentrations (X)
Normalize
Prune rows
Remove baseline and column math
Transpose X and Y
Fraction of Total
XY analyses
Column analyses
Grouped analyses
Contingency table analyses
Survival analyses
Parts of whole analyses
Fraction of Total
Compare observed distribution with ex…
Multiple variable analyses
Nested analyses
Generate curve
Simulate data
Recently used
A:year 1634
B:total
Select All
Deselect All
?
Cancel
OK


Parameters: Compare observed distribution with expected
This analysis expects that each value in the data table is an actual number of events or items, and is not normalized in any way.
Data set to analyze
A: year 1634
Enter expected values as
Actual numbers of objects or events
Percentages
o rows, perform
nomial test (recommended)
-square test for goodness of fit
Expected distribution
Row
Outcome
Observed %
Expected %
1
boys
51.09
50
2
girls
48.91
50
Output
Method to calculate CI:
Wilson/Brown (recommended)
Show this many significant digits (for everything except P values):
4
P value style:
GP: 0.1234 (ns), 0.0332 (*), 0.0021 (**), 0.000...
N=
6
?
Cancel



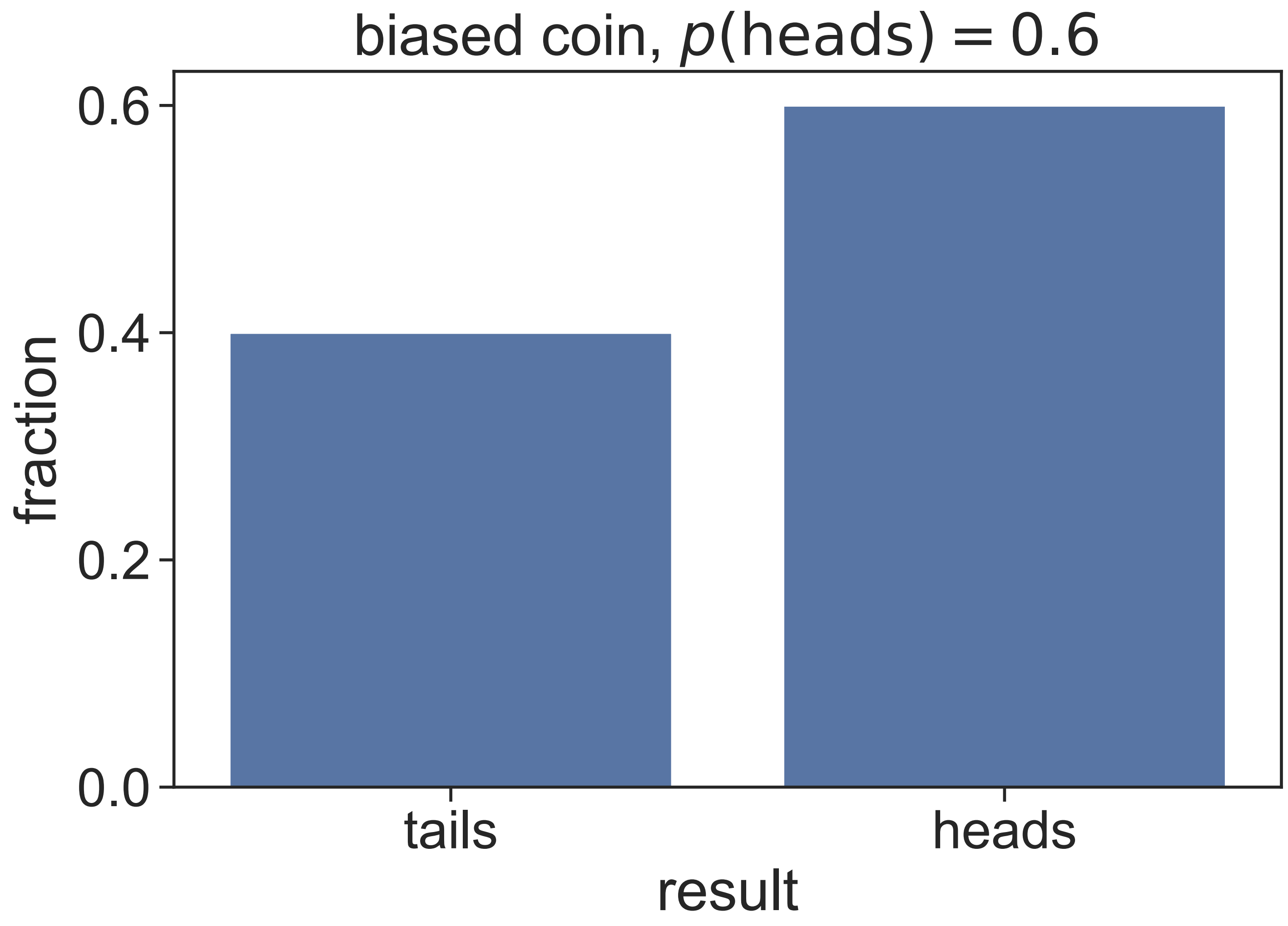
	O vs. E					
1	Table analyzed	Data 1				
2	Column analyzed	Column A				
3						
4	**Binomial test**					
5	P (one-tailed)	0.0156				
6	P (two-tailed)	0.0311				
7	P value summary	*				
8	Is discrepancy significant (P < 0.05)?	Yes				
9						
10	**Outcome**	**Expected #**	**Observed #**	**Expected %**	**Observed %**	**95% CI of Observed %**
11	boys	4928	5035	50.00	51.09	50.10 to 52.08
12	girls	4928	4820	50.00	48.91	47.92 to 49.90
13	TOTAL	9855	9855	100.0	100.00	

Births in London, 1629-1710



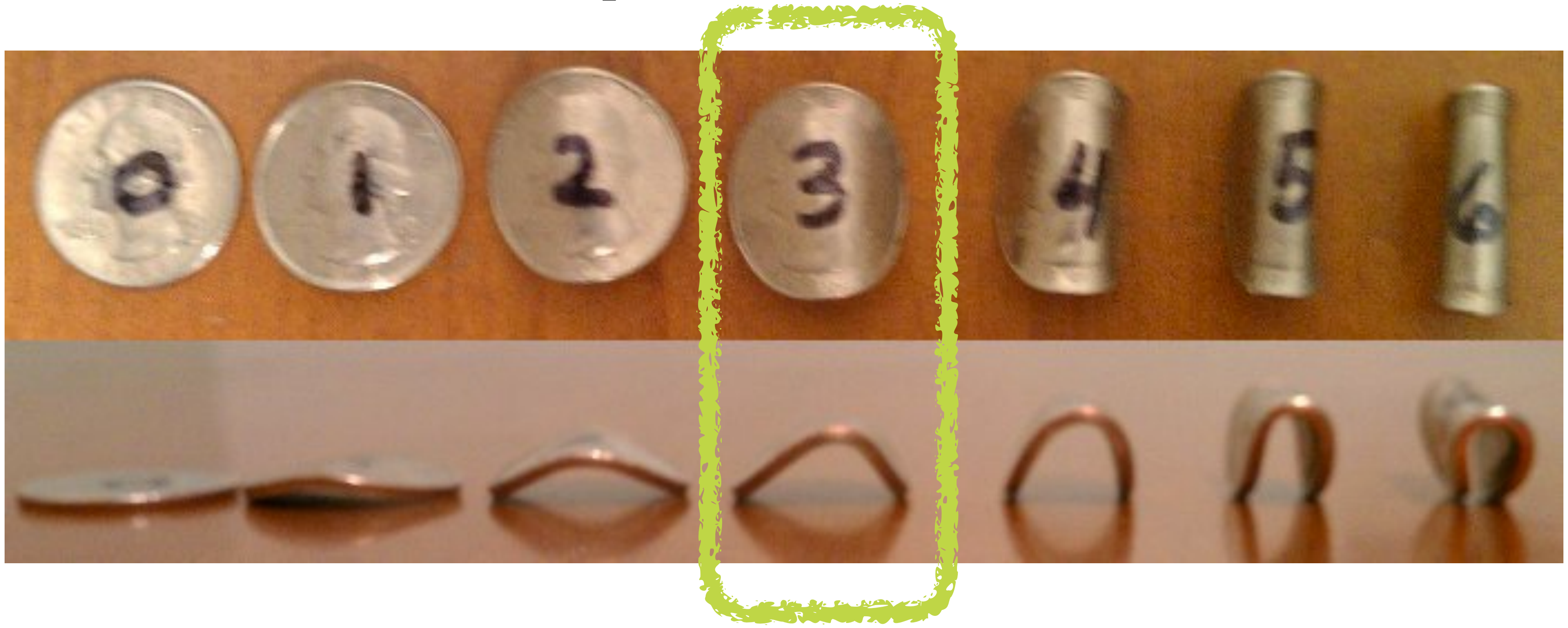
Example 2: A biased coin

Biased coins are modeled using a Bernoulli distribution, which describes probabilities for a binary variable



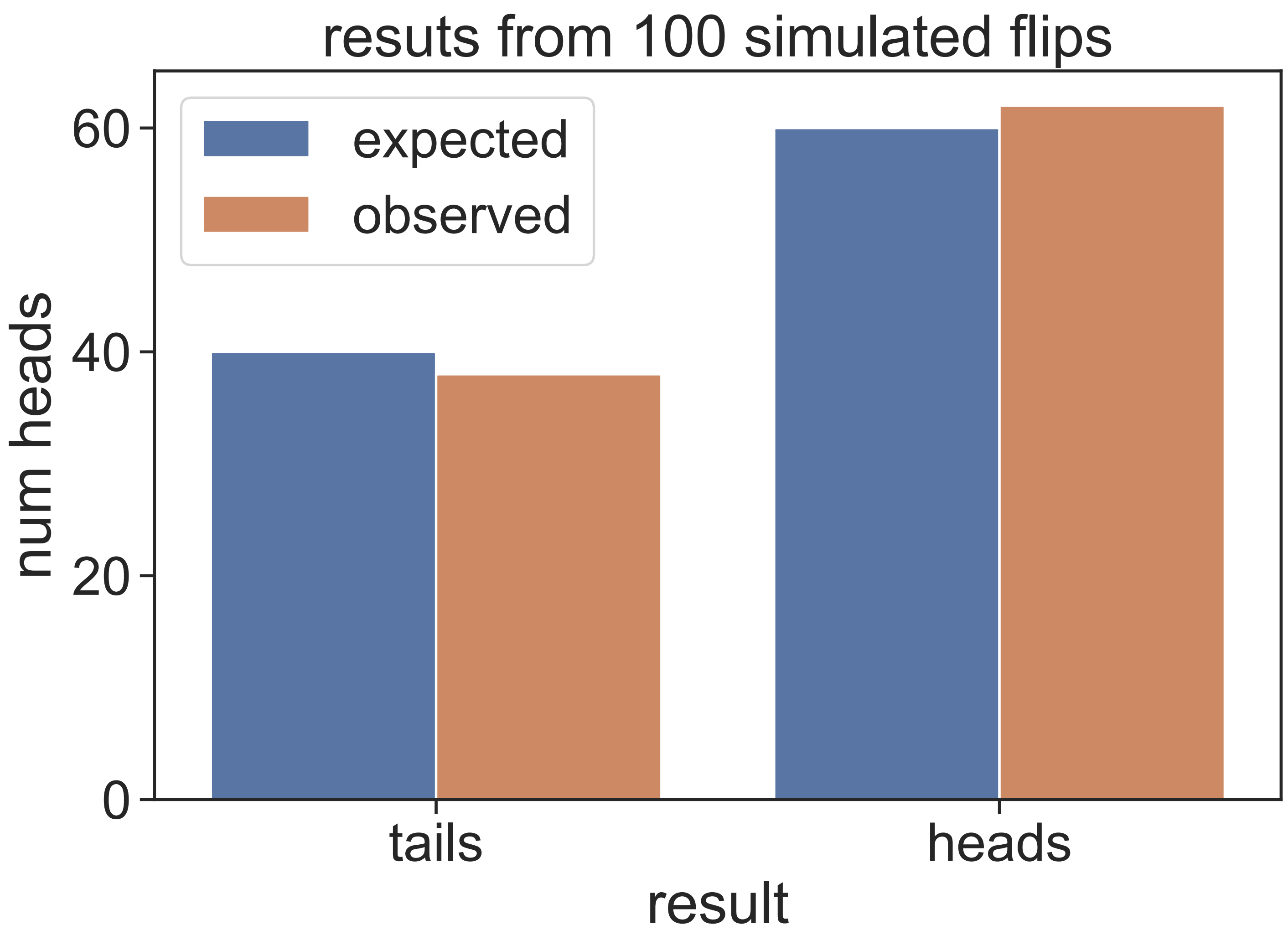
Making a biased coin

$p(\text{heads}) \approx 60\,\%$



Mike Izbicki (Claremont McKenna College)
https://izbicki.me/blog/how-to-create-an-unfair-coin-and-prove-it-with-math.html

The number of heads after 100 flips of the biased coin will resemble the underlying probabilities, but will not match exactly



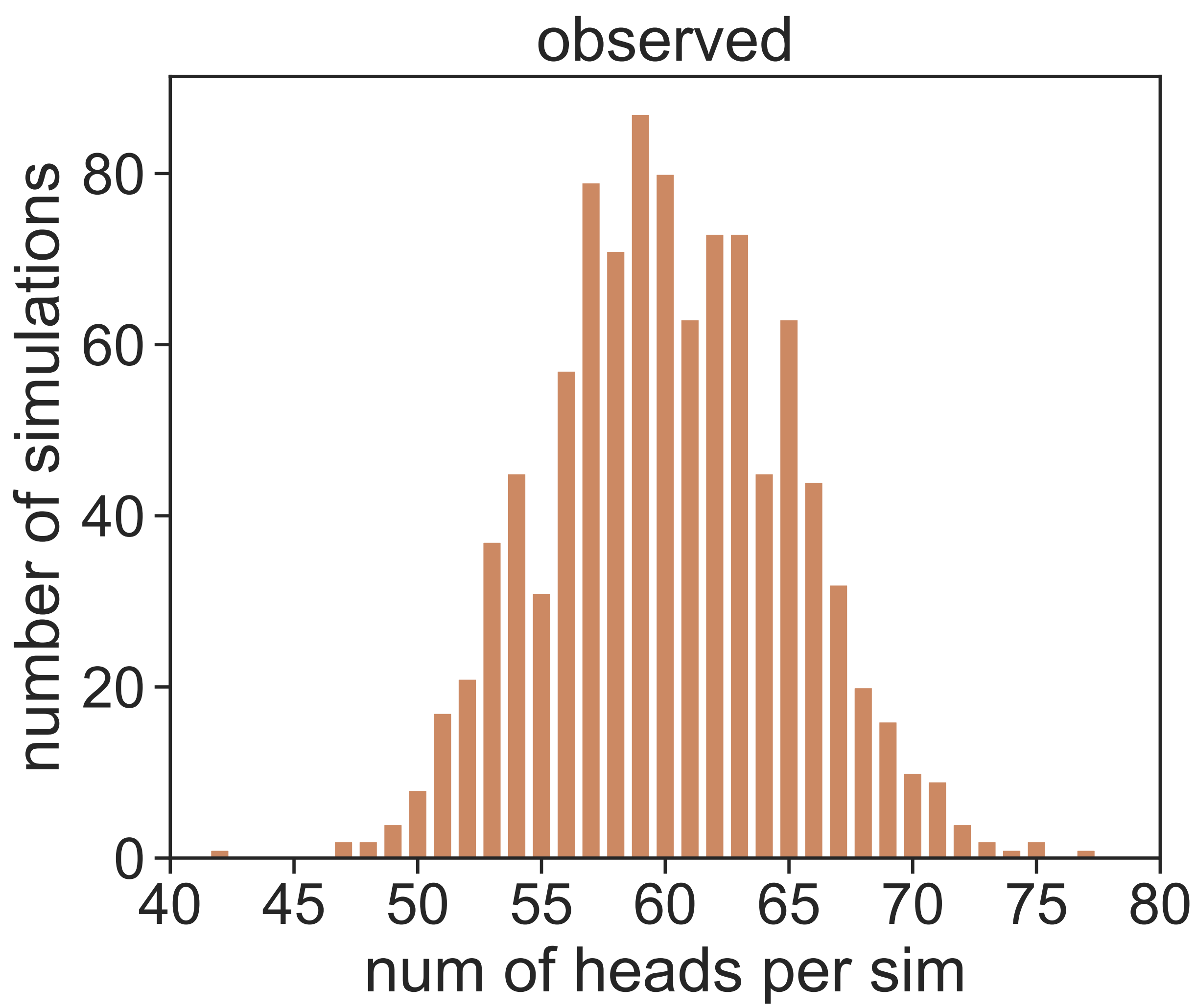
expected: 60 heads, 40 tails

observed: 62 heads, 38 tails

How much deviation from the expected values do we expect?

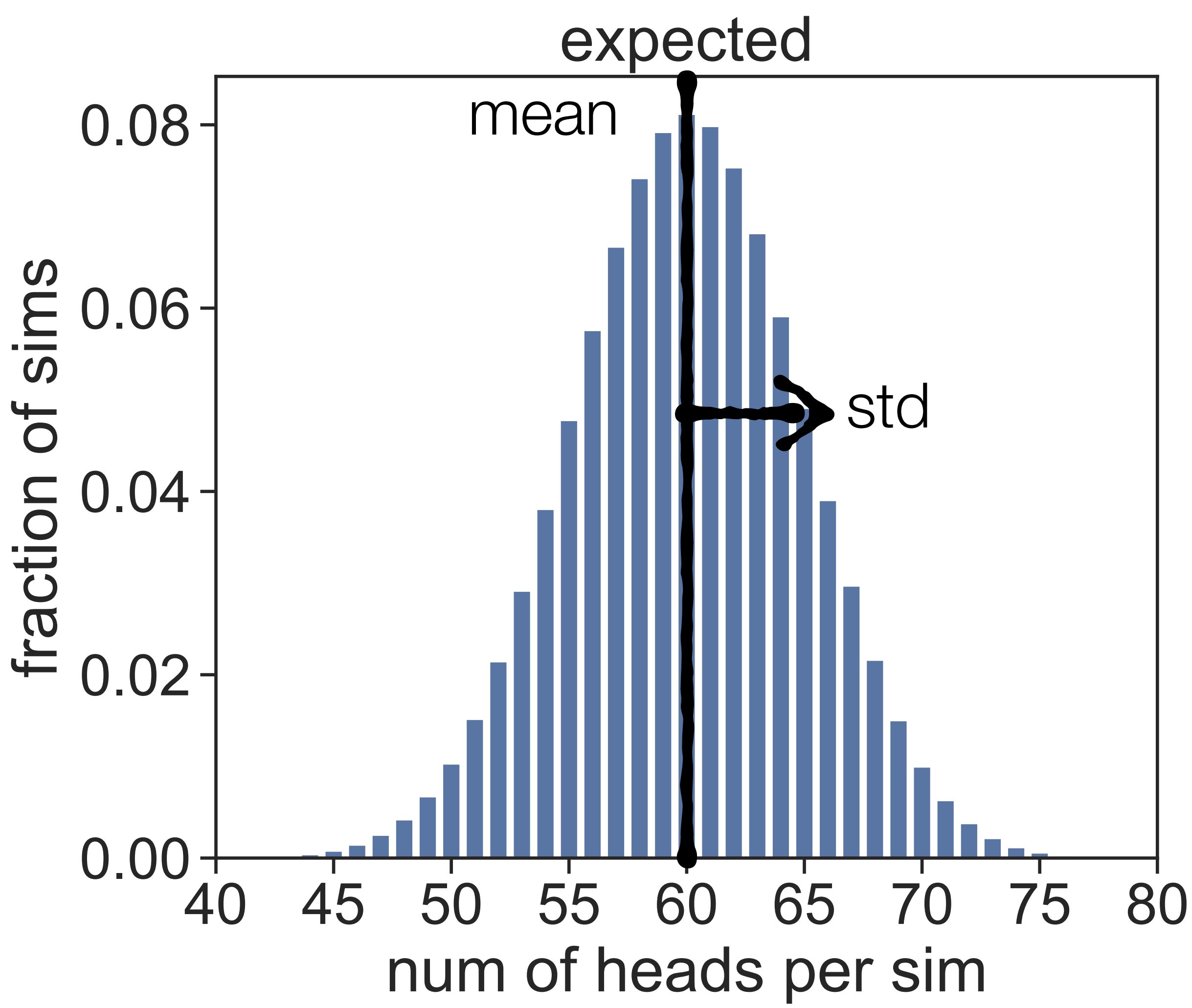
There is substantial variation across replicates. This is to be expected.

Results from 1000 simulations, 100 flips per simulation



The variation in the number of heads from replicate to replicate is described by a binomial distribution

Results from 1000 simulations, 100 flips per simulation



mean = 60

standard deviation (std) = 4.9

Can we determine whether or not a coin is biased by flipping it 100 times?

Suppose we flip a coin 100 times and observe **62 heads** (and 38 tails).

Null hypothesis: heads and tails are equally likely, i.e.

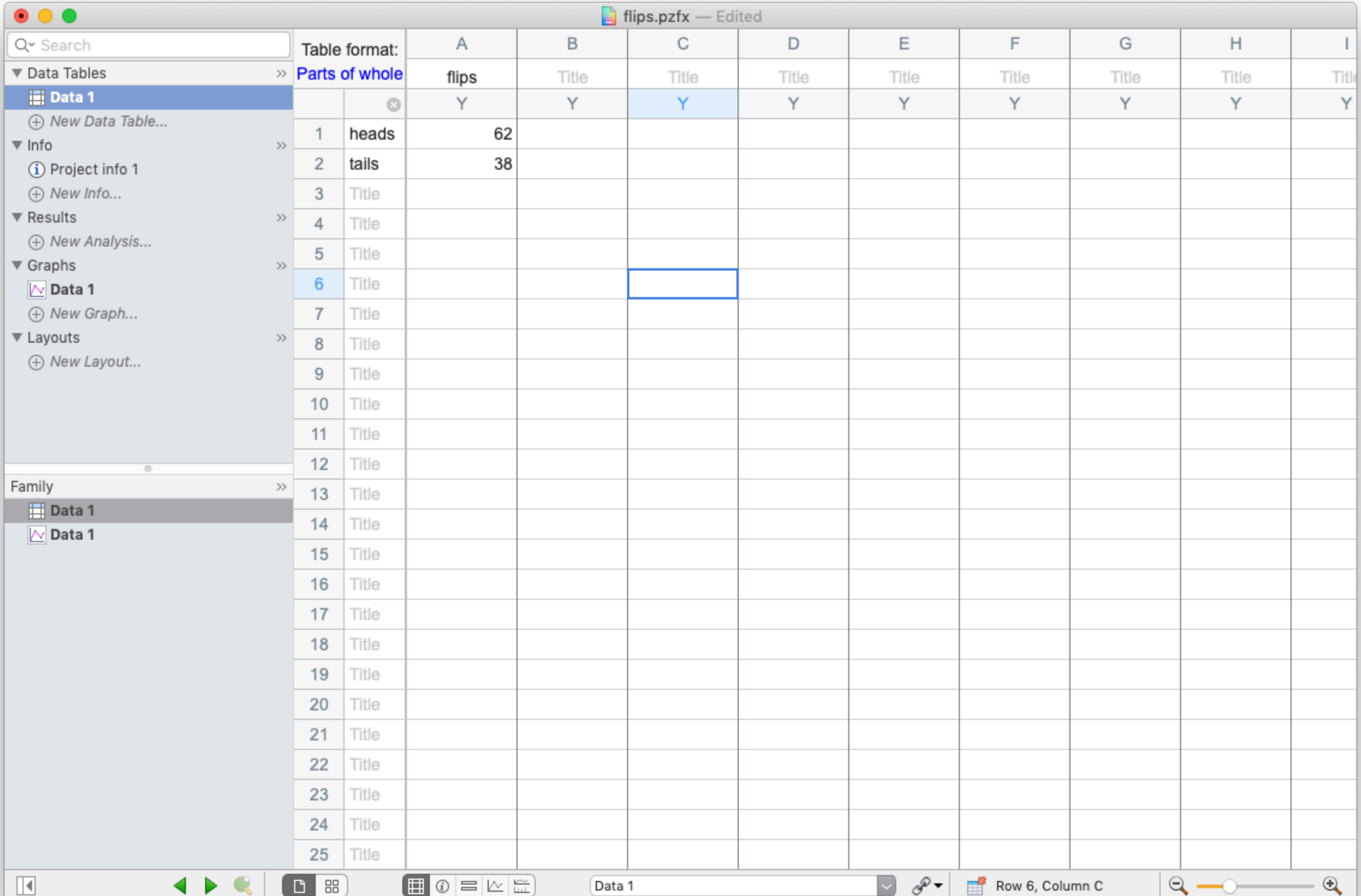
$$p(\text{heads}) = 50\,\%$$

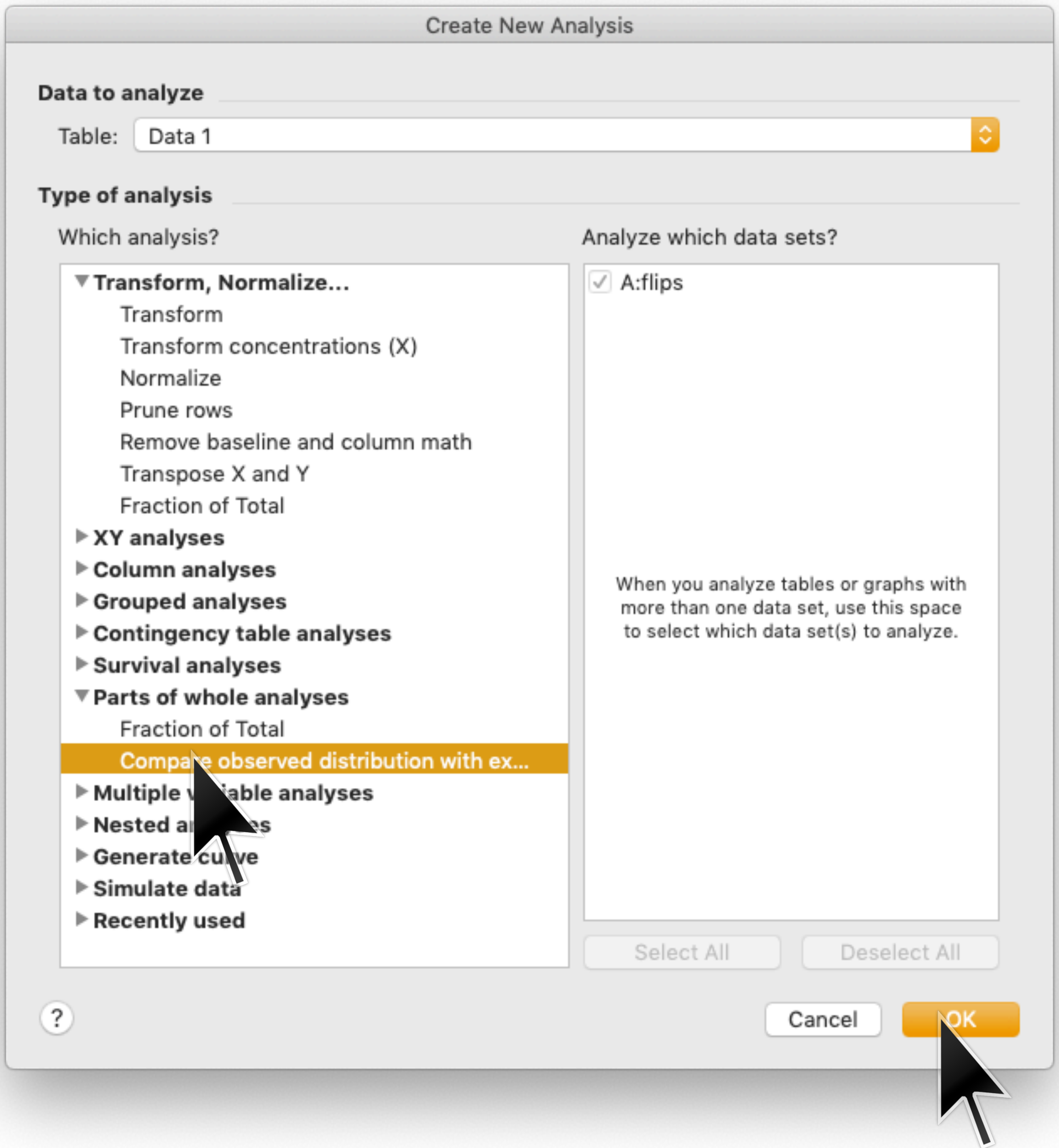
Alternative hypothesis: heads and tails are not equally likely, i.e.

$$p(\text{heads}) \neq 50\,\%$$

Our observation (62 heads) may or may not allow us to <u>reject the null hypothesis</u> and thus <u>accept the alternative hypothesis</u>.

No amount of data, however, can cause us to <u>accept the null hypothesis</u>.


flips.pzfx — Edited
Search
Data Tables
Data 1
New Data Table...
Info
Project info 1
New Info...
Results
New Analysis...
Graphs
Data 1
New Graph...
Layouts
New Layout...
Family
Data 1
Data 1
Table format:
Parts of whole
A
B
C
D
E
F
G
H
I
flips
Title
Y
1 heads 62
2 tails 38
Data 1
Row 6, Column C


Create New Analysis
Data to analyze
Table:
Data 1
Type of analysis
Which analysis?
Transform, Normalize...
Transform
Transform concentrations (X)
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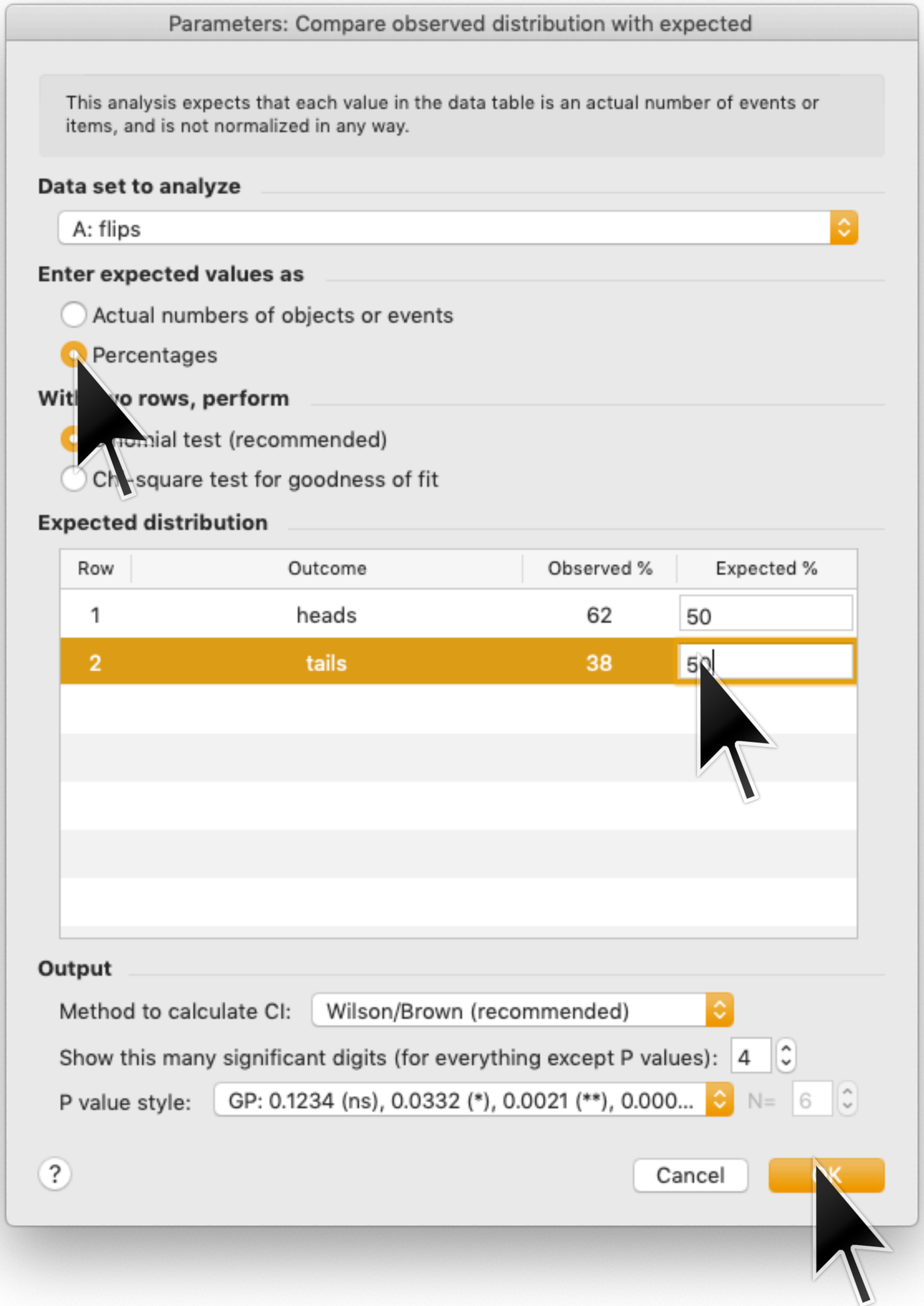
Prune rows
Remove baseline and column math
Transpose X and Y
Fraction of Total
XY analyses
Column analyses
Grouped analyses
Contingency table analyses
Survival analyses
Parts of whole analyses
Fraction of Total
Compa e observed distribution with ex...
Multiple v iable analyses
Nested a es
Generate cu ve
Simulate data
Recently used
Analyze which data sets?
A:flips
When you analyze tables or graphs with more than one data set, use this space to select which data set(s) to analyze.
Select All
Deselect All
?
Cancel
OK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This analysis expects that each value in the data table is an actual number of events or items, and is not normalized in any way.
Data set to analyze
A: flips
Enter expected values as
Actual numbers of objects or events
Percentages
rows, perform
test (recommended)
square test for goodness of fit
Expected distribution
Row
Outcome
Observed %
Expected %
1
heads
62
50
2
tails
38
Output
Method to calculate CI:
Wilson/Brown (recommended)
Show this many significant digits (for everything except P values):
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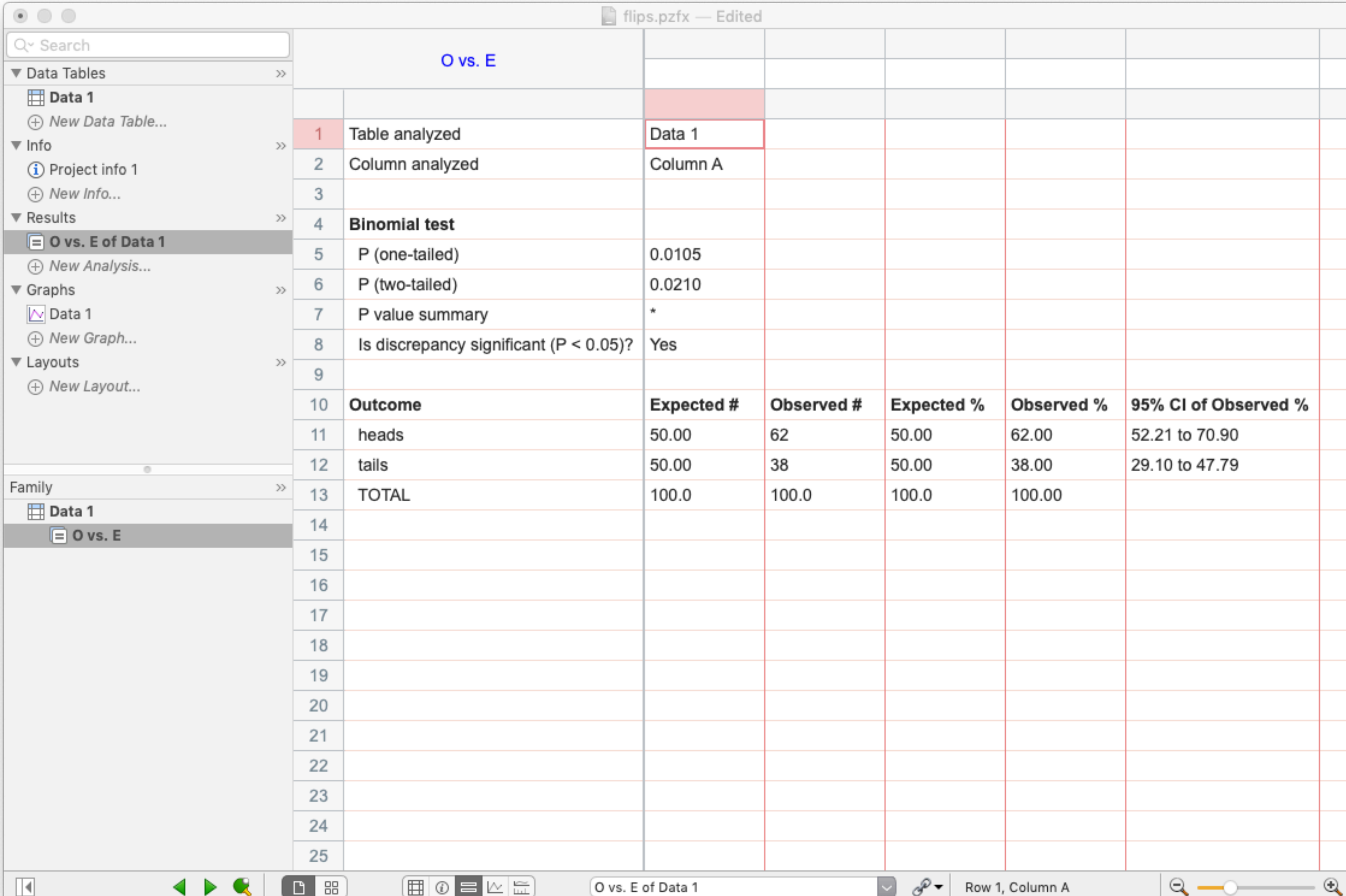
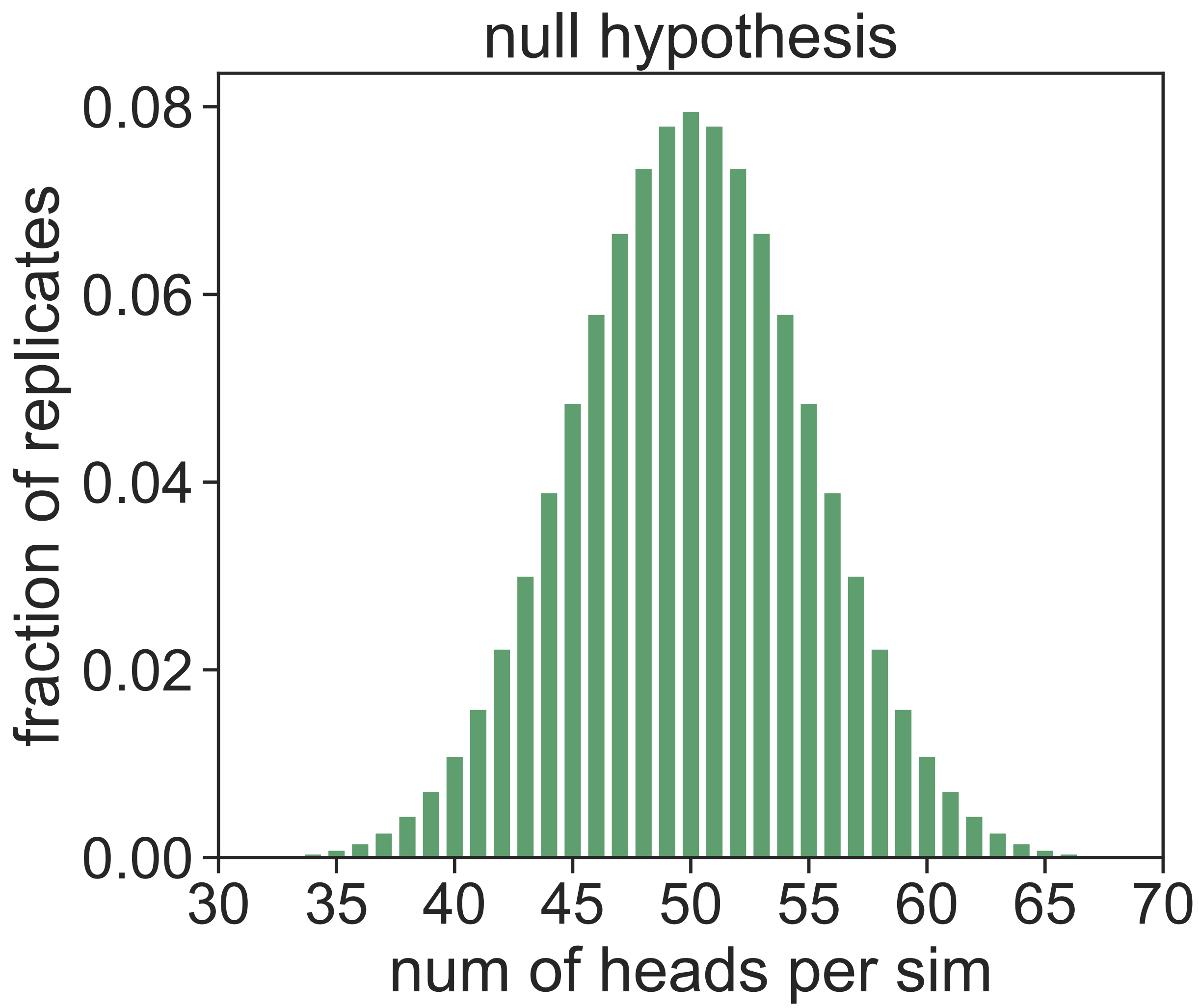
P value style:
GP: 0.1234 (ns), 0.0332 (*), 0.0021 (**), 0.000...
N=
6
?
Cancel

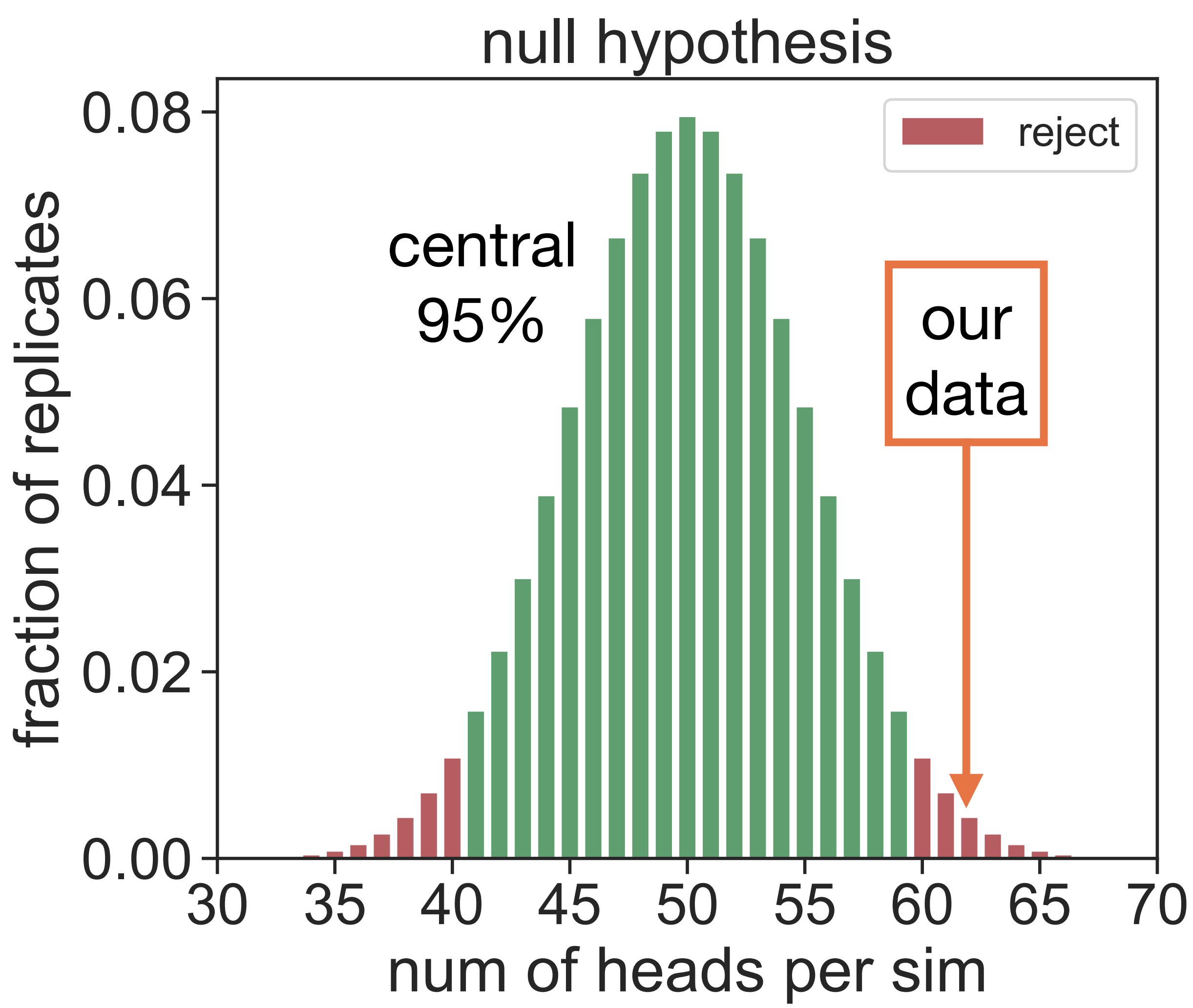

flips.pzfx — Edited
Search
Data Tables
Data 1
New Data Table...
Info
Project info 1
New Info...
Results
O vs. E of Data 1
New Analysis...
Graphs
Data 1
New Graph...
Layouts
New Layout...
Family
Data 1
O vs. E
O vs. E
1 Table analyzed Data 1
2 Column analyzed Column A
3
4 Binomial test
5 P (one-tailed) 0.0105
6 P (two-tailed) 0.0210
7 P value summary *
8 Is discrepancy significant (P < 0.05)? Yes
9
10 Outcome Expected # Observed # Expected % Observed % 95% CI of Observed %
11 heads 50.00 62 50.00 62.00 52.21 to 70.90
12 tails 50.00 38 50.00 38.00 29.10 to 47.79
13 TOTAL 100.0 100.0 100.0 100.00
O vs. E of Data 1
Row 1, Column A

The null hypothesis is assessed by where the data fall within the null distribution



We reject the null hypothesis of the data fall too far away from the bulk of the distribution

If the null hypothesis is true, data should fall within the green region 95% of the time, and within the red "reject" region 5% of the time.

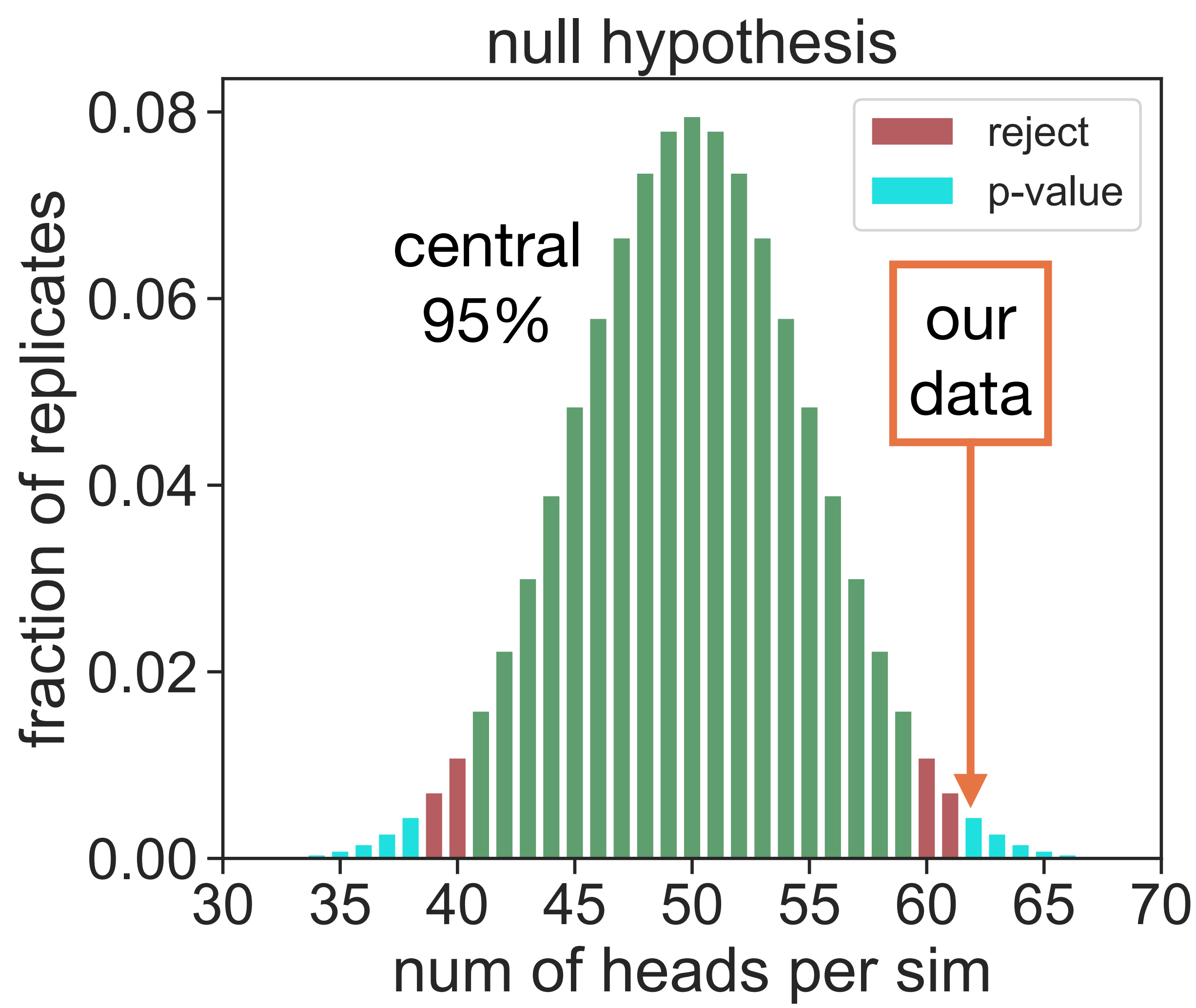


Our assumed dataset (62 heads) lies outside the central 95%.

We can therefore <u>reject the null hypothesis</u> with <u>95% confidence</u>.

P-values quantify the probability of data being as or more extreme than the data in hand were the null hypothesis true.

The P-value threshold of 0.05 comes from adopting a confidence threshold of 95%.

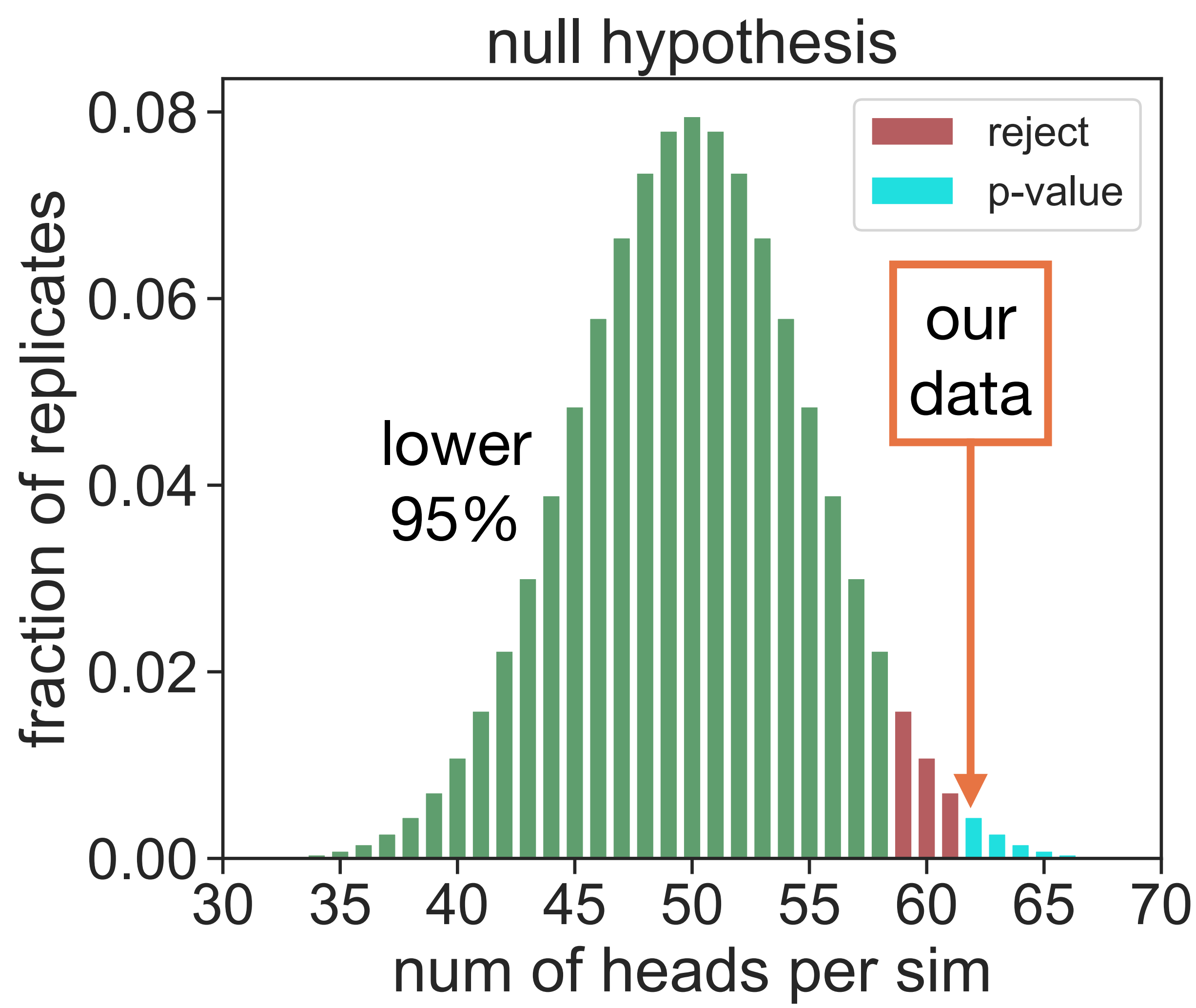


We find that **p=0.0210** for the two-sided test.

We therefore say that our result is "statistically significant"

P-values quantify the probability of data being as or more extreme than the data in hand were the null hypothesis true.

A one-sided hypothesis test only considers one side of the distribution.

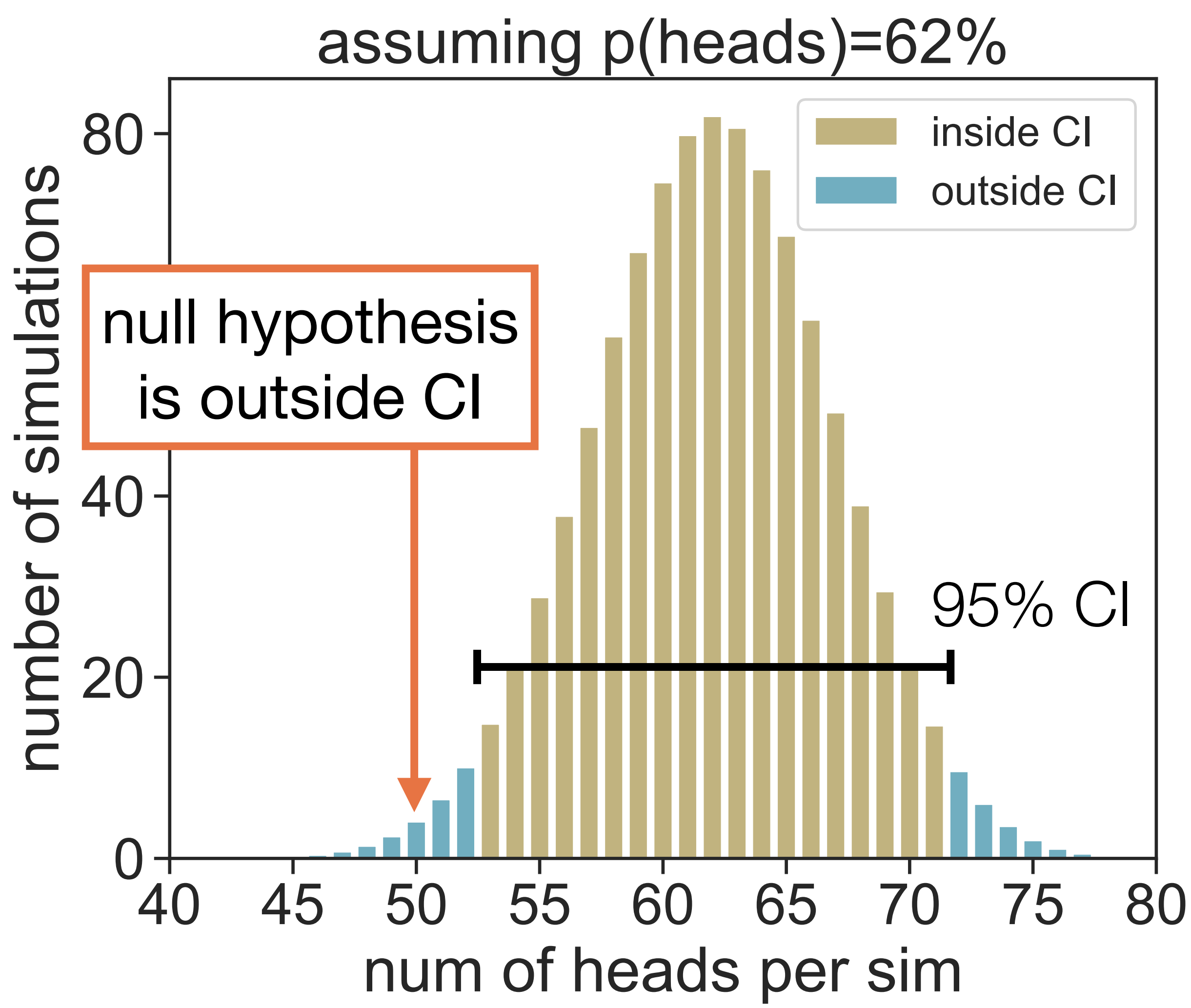


We find that **p=0.0105** for the one-sided test.

In general, two-sided tests are more conservative than one-sided tests.

Unless you have good reason to do otherwise, use two-sided tests.

Confidence intervals (CIs) are more informative than P-values



We conclude that $p(\text{heads})$ lies within [52.5%, 71.5%] with 95% confidence.

We can reject the null hypothesis because it lies outside this confidence interval.

P-values have multiple pitfalls

- “Statistically significant” does not actually mean “significant” in the normal sense. At best, it means “detectable”.
- P-values do not say how big an observed effect is.
- P-values do not say how important that observed effect is.
- P-values calculations rely on assumptions, and violation of any of those assumptions can render P-values meaningless.
- Perhaps most severe is the fact that <u>P-values do not actually quantify you how likely or unlikely your null hypothesis is</u>!

Why are Confidence Intervals better than P-values?

- Like a P-value, a CI communicates statistical significance (i.e. detectability).
- A CI also communicates effect size, as well as the uncertainty in that effect size.
- A 95% CI does not actually mean that the true value of a parameter lies within that interval with 95% probability. Still, this (extremely common) misinterpretation is largely benign compared to the misinterpretation of P-values.
- However, P-values are more commonly reported than confidence intervals.

The perils of null hypothesis testing

Summary of null hypothesis testing

Step 1: Specify a null hypothesis.

Step 2: Specify a confidence level (usually 95%)

Step 3: Identify the appropriate statistical test

Then:
evaluate on data

Result: P-value summarizing how unlikely the data is compared to null hypothesis expectations.

Perhaps most problematic is how easily P-values are misinterpreted.

Roughly speaking, P-values quantify how likely our data would be if the null hypothesis were true.

$$p(\text{data} \,|\, \text{null hypothesis})$$

P-values <u>do not</u> quantify the probability of the null hypothesis given our data. Unfortunately, this is the quantity that we actually care about.

$$p(\text{null hypothesis} \,|\, \text{data})$$

My opinion: the use of P-values to reject hypotheses is predicated on the base rate fallacy

By convention $P < 0.05$, then one rejects null hypothesis, supposedly because $p(\text{null hypothesis} \mid \text{data})$ is small.

For this to make sense, one has to accept the base rate fallacy, i.e.,

$$p(\text{data} \mid \text{null hypothesis}) \approx p(\text{null hypothesis} \mid \text{data})$$

Whether or not this is true in a specific case depends on the prior odds,

$$p(\text{null hypothesis}),$$

which Frequentist statistics refuses to consider.

The misinterpretation of P-values reflects the Frequentist / Bayesian divide

Frequentist statistics (a.k.a. classical statistics) focuses on likelihood:

$$p(\text{data} \mid \text{hypothesis}).$$

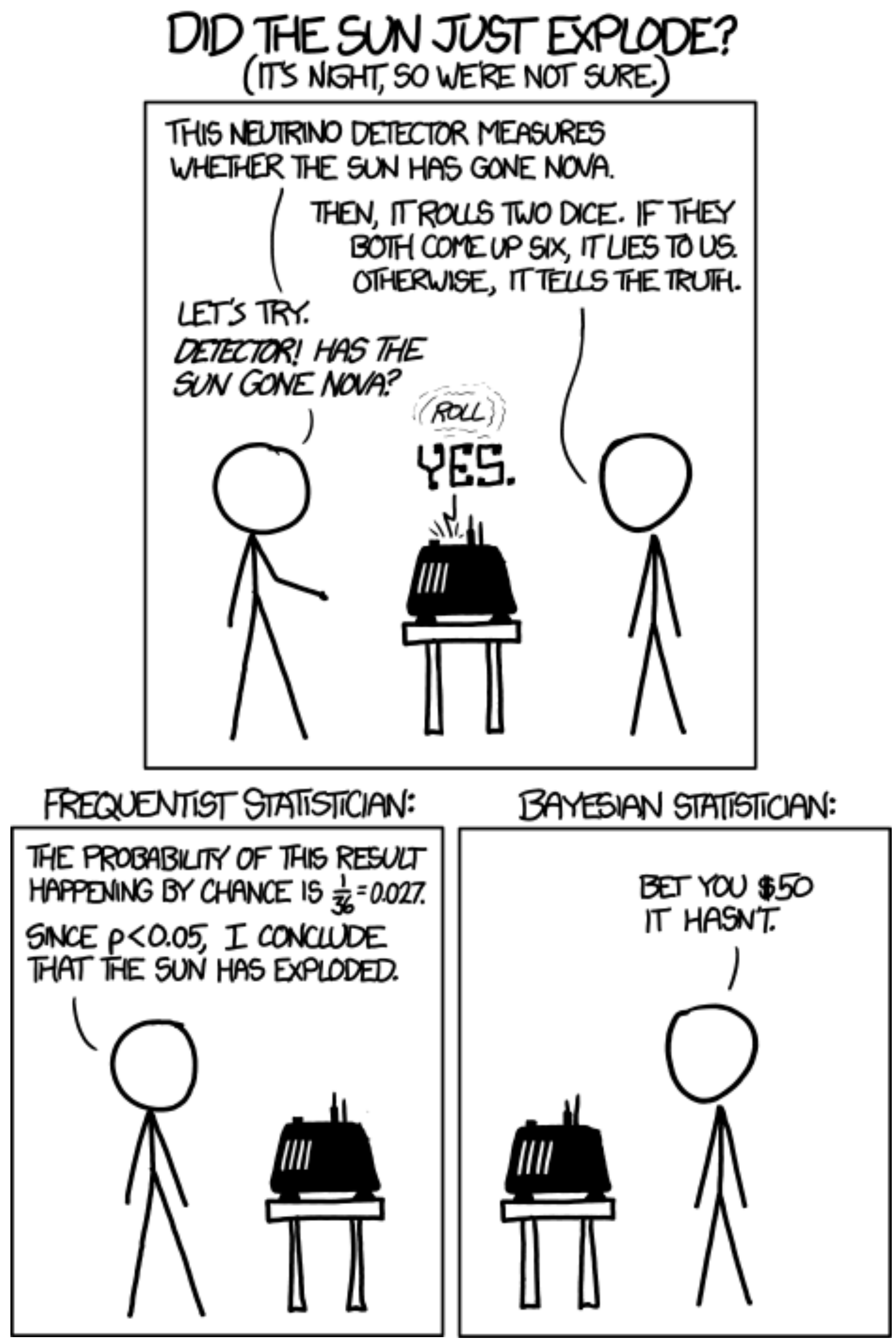
Iron Law of Frequentist Statistics:
Never compute the probability of a hypothesis.

Bayesian statistics focuses on computing posterior probabilities:

$$p(\text{hypothesis} \mid \text{data}).$$

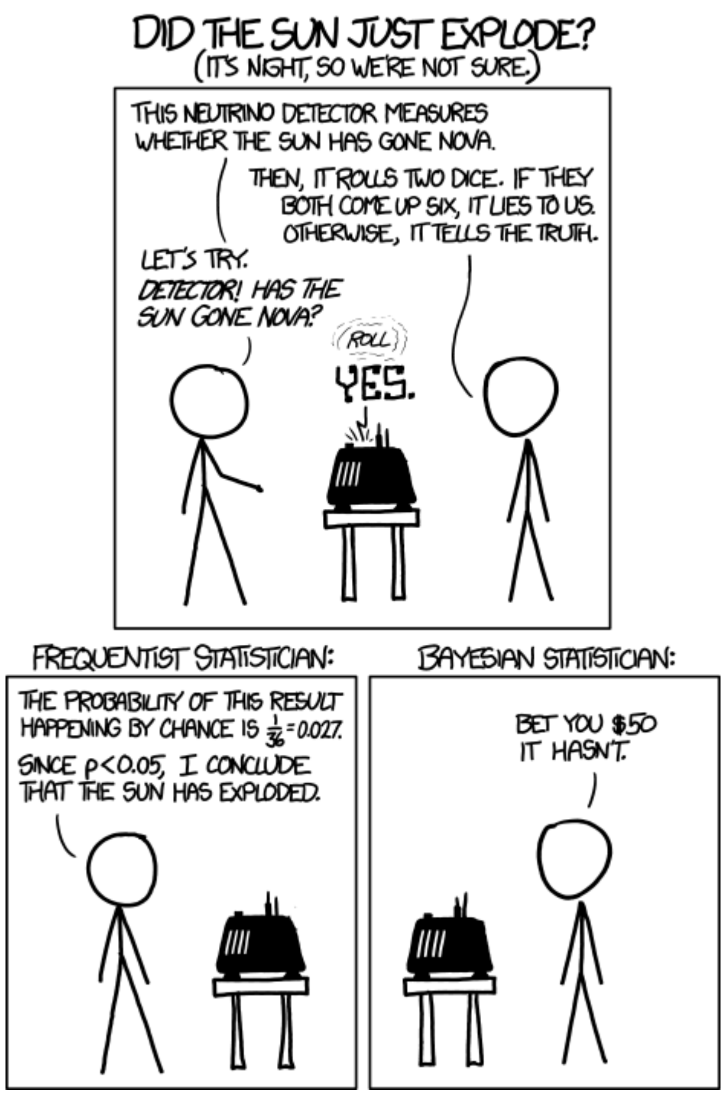
Example 3: Supernova detection machine

Exercise: supernova



https://xkcd.com/1132/

Exercise: supernova



https://xkcd.com/1132/

Bayes's theorem (from yesterday):

$$\frac{p(\mathrm{nova}^+\,|\,\mathrm{detector}^+)}{p(\mathrm{nova}^-\,|\,\mathrm{detector}^+)} = \frac{p(\mathrm{detector}^+\,|\,\mathrm{nova}^+)}{p(\mathrm{detector}^+\,|\,\mathrm{nova}^-)} \times \frac{p(\mathrm{nova}^+)}{p(\mathrm{nova}^-)}$$

$$\left[\frac{35/36}{1/36} = 35\right]$$

If our prior belief is that a supernova is very unlikely, i.e.

$$\frac{p(\mathrm{nova}^+)}{p(\mathrm{nova}^-)} \ll \frac{1}{35},$$

then we still shouldn't believe the sun has gone nova.

Even though, with a null hypothesis of nova^-,

$$\mathrm{P\ value} = p(\mathrm{detector}^+\,|\,\mathrm{nova}^-) = \frac{1}{36} = 0.028 < 0.05$$

Example 4: Mendel's Peas

	Flower Colour	Plant Height	Seed Color	Seed Shape	Pod Colour	Pod Shape	Flower Position
Dominant Trait	Purple	Tall	**3/4** Yellow	**3/4** Round	Green	Inflated (full)	Axial
Recessive Trait	White	Short	**1/4** Green	**1/4** Wrinkled	Yellow	Constricted (flat)	Terminal

https://ib.bioninja.com.au/standard-level/topic-3-genetics/34-inheritance/mendels-laws.html

Chi square test (known proportions)

Example: Mendel's peas

	observed	expected proportion	expected counts
Round & yellow	315	9/16	312.75
Round & green	108	3/16	104.25
Angular & yellow	101	3/16	104.25
Angular & green	32	1/16	34.75
Total	556	16/16	556.00

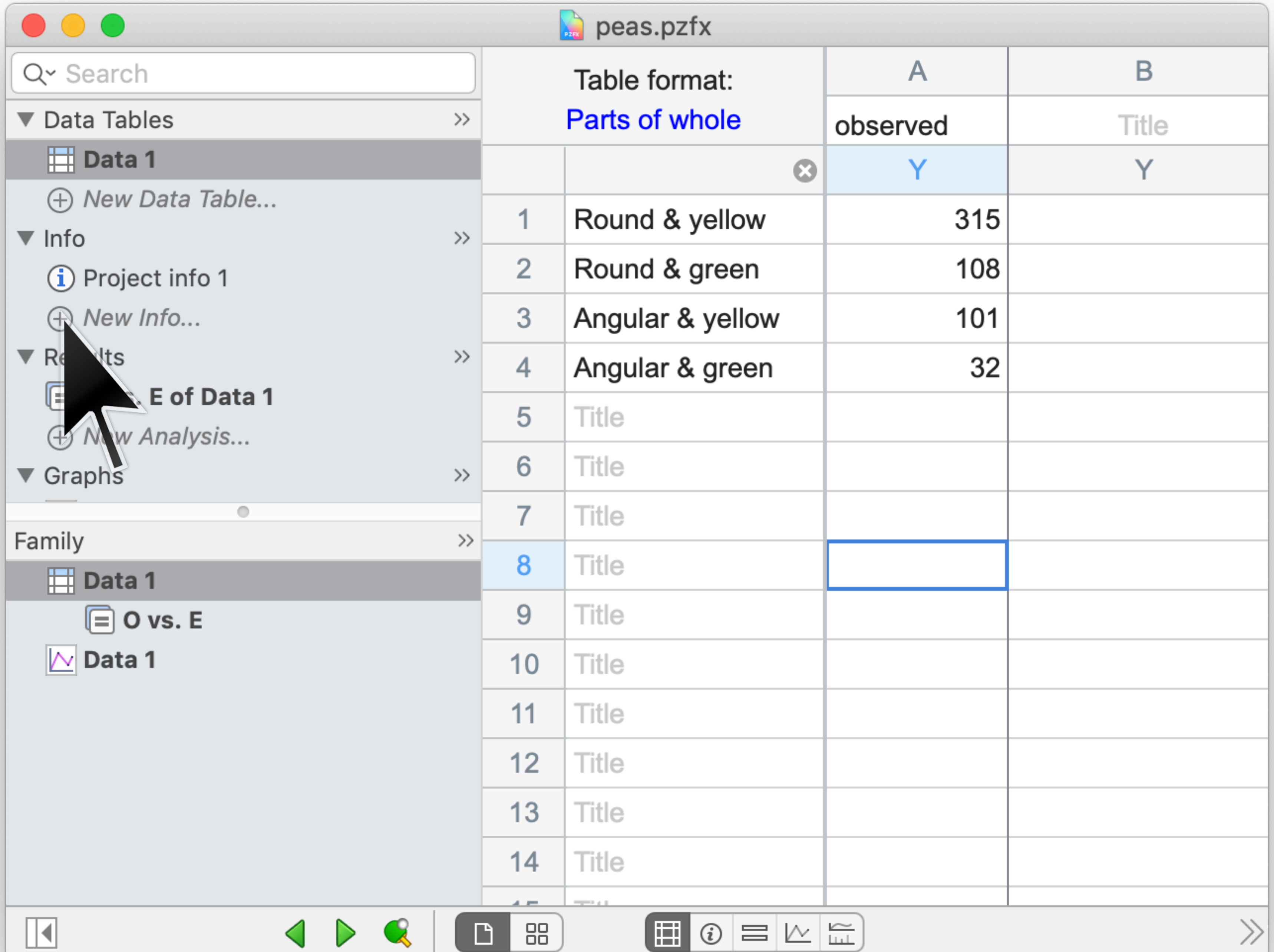
Null Hypothesis:

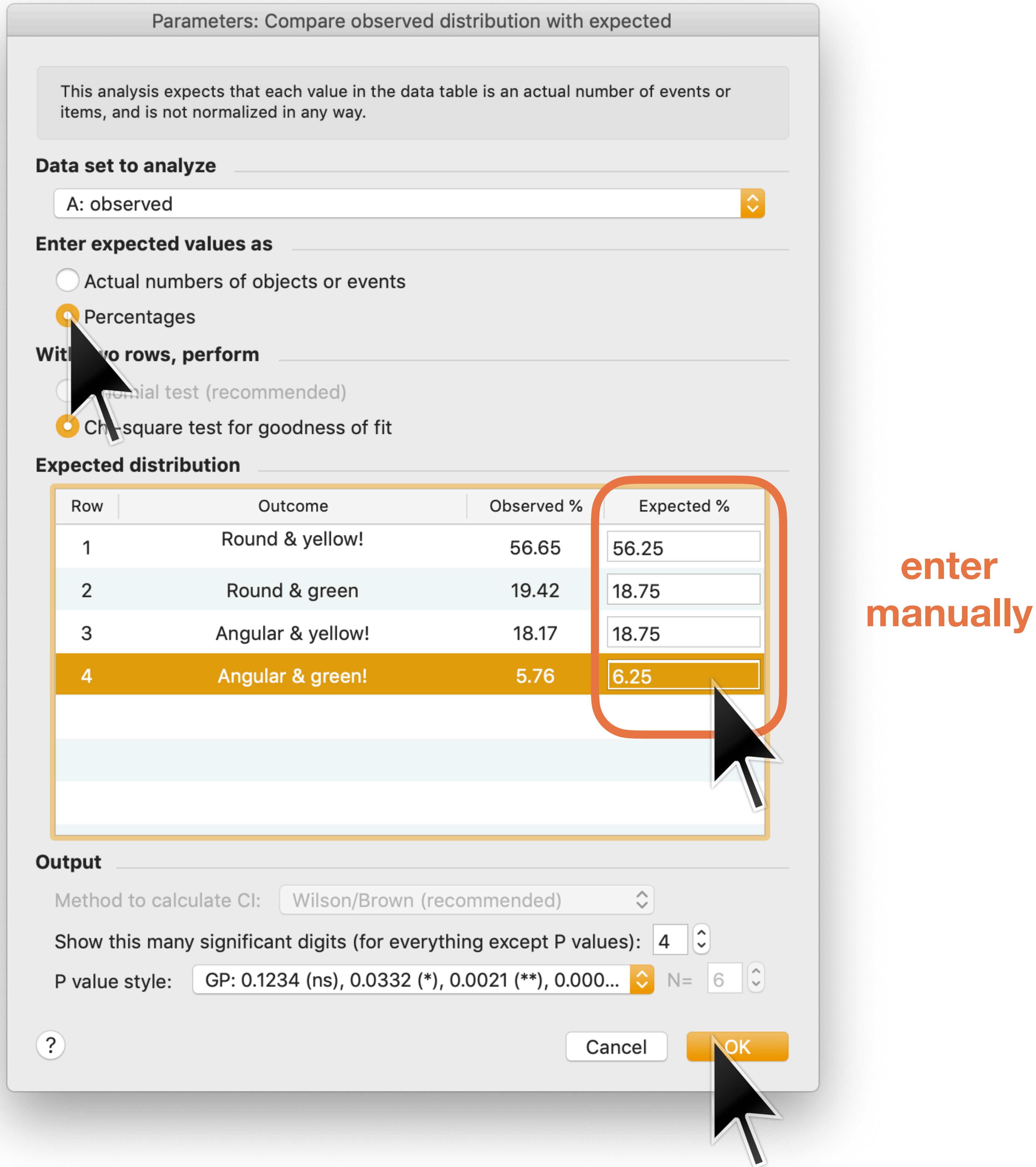
observations in $K = 4$ different categories occur in the expected proportions

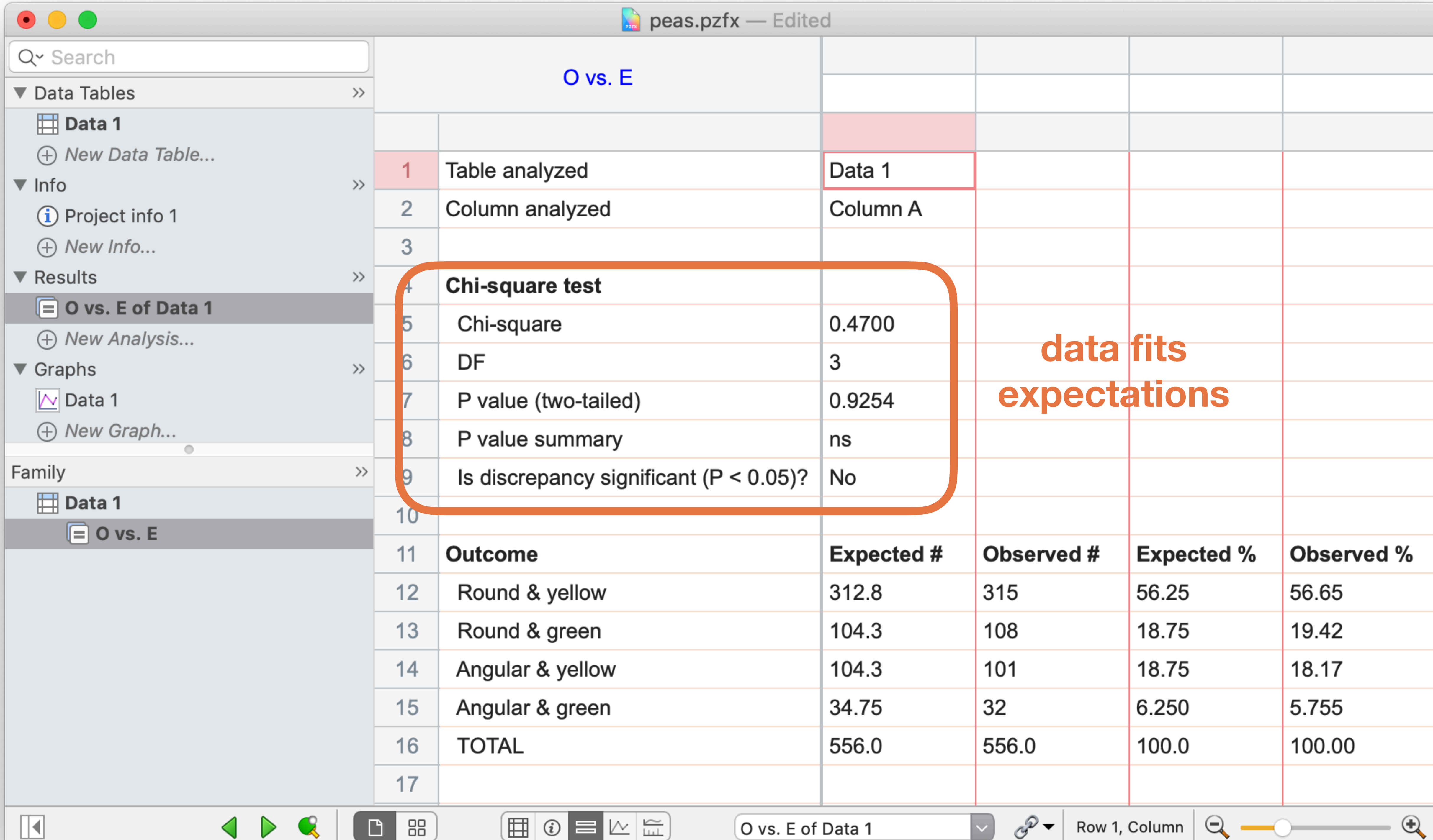
Data: number of observations in each category

Statistic: $\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$

Null distribution: Chi square distribution with $K - 1 = 3$ degrees of freedom (DOF)


peas.pzfx
Search
Data Tables
Data 1
New Data Table...
Info
Project info 1
New Info...
E of Data 1
Graphs
Family
Data 1
O vs. E
Data 1
Table format:
Parts of whole
A
B
observed
Title
Y
Y
1 Round & yellow 315
2 Round & green 108
3 Angular & yellow 101
4 Angular & green 32
5 Title
6 Title
7 Title
8 Title
9 Title
10 Title
11 Title
12 Title
13 Title
14 Title

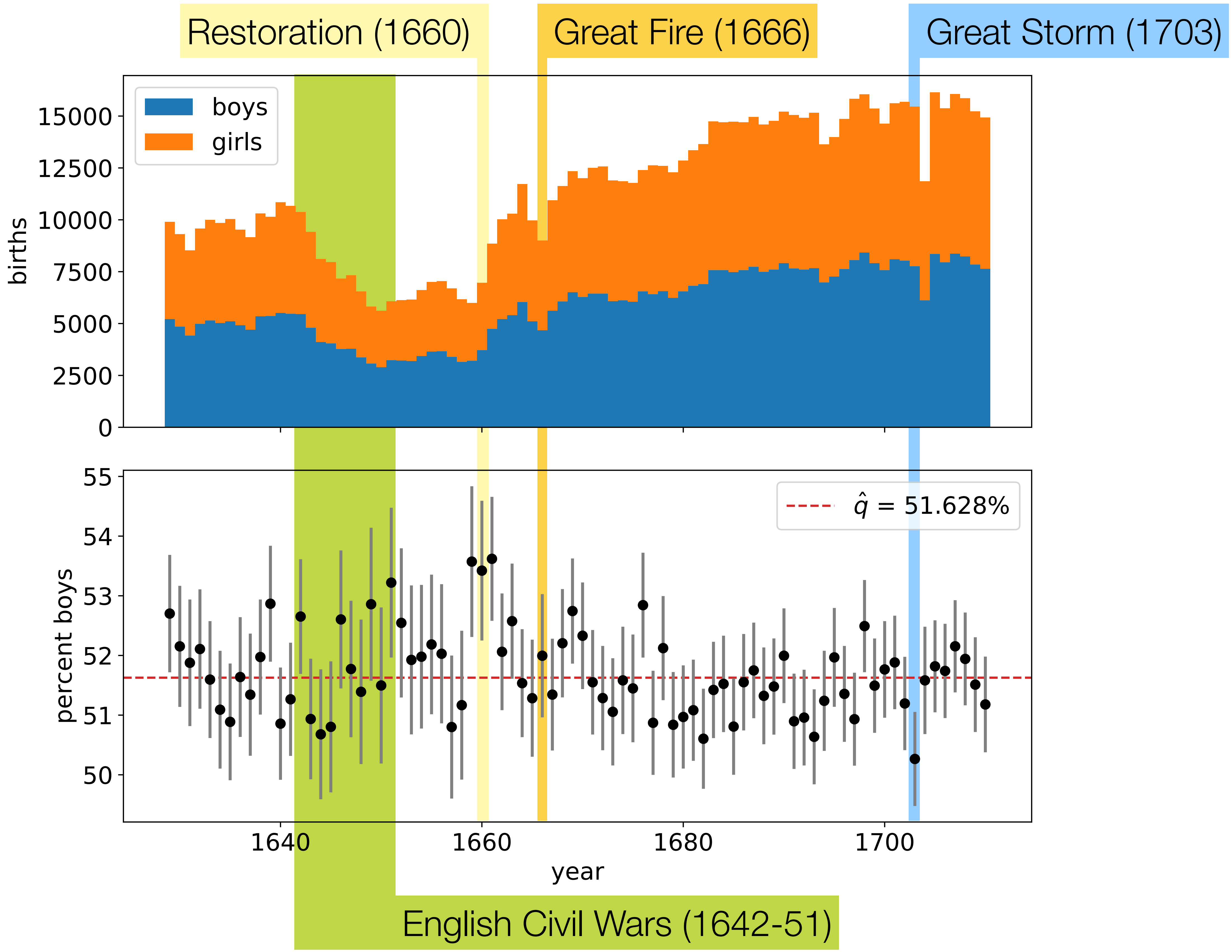

Parameters: Compare observed distribution with expected
This analysis expects that each value in the data table is an actual number of events or items, and is not normalized in any way.
Data set to analyze
A: observed
Enter expected values as
Actual numbers of objects or events
Percentages
Expected distribution
Row
Outcome
Observed %
Expected %
1 Round & yellow! 56.65 56.25
2 Round & green 19.42 18.75
3 Angular & yellow! 18.17 18.75
4 Angular & green! 5.76 6.25
Output
Method to calculate CI: Wilson/Brown (recommended)
Show this many significant digits (for everything except P values): 4
P value style: GP: 0.1234 (ns), 0.0332 (*), 0.0021 (**), 0.000...
N= 6
?
Cancel
OK
enter manually



1	Table analyzed	Data 1			
2	Column analyzed	Column A			
3					
4	**Chi-square test**				
5	Chi-square	0.4700			
6	DF	3			
7	P value (two-tailed)	0.9254			
8	P value summary	ns			
9	Is discrepancy significant (P < 0.05)?	No			
10					
11	**Outcome**	**Expected #**	**Observed #**	**Expected %**	**Observed %**
12	Round & yellow	312.8	315	56.25	56.65
13	Round & green	104.3	108	18.75	19.42
14	Angular & yellow	104.3	101	18.75	18.17
15	Angular & green	34.75	32	6.250	5.755
16	TOTAL	556.0	556.0	100.0	100.00
17					

Example 4: Human sex ratio in London over time

Is it possible that the boy/girl ratio changes from year to year?



Chi square test (unknown proportions)

sex

year

	male	female
1629	5218	4683
1630	4858	4457
1631	4422	4102
1632	4994	4590
1633	5158	4839
1634	5035	4820
1635	5106	4928
1636	4917	4605
1637	4703	4457
1638	5359	4952

Null Hypothesis:

Two multi-category variables A and B are independent, i.e.,

$$p(A, B) = p(A) \cdot p(B)$$

Statistic:

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

Null distribution:

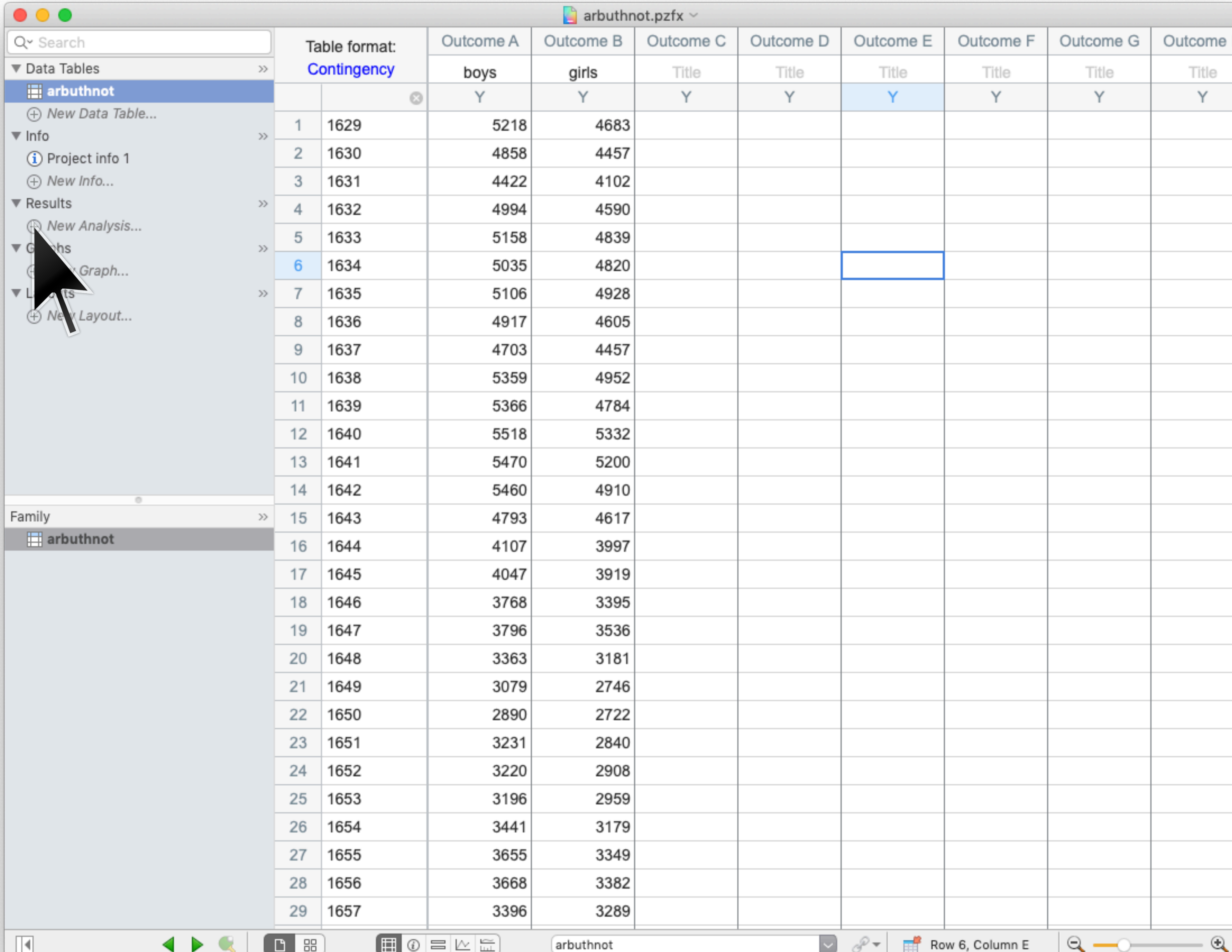
Chi square distribution with

$$\text{DOF} = nm - m - n + 1$$

where

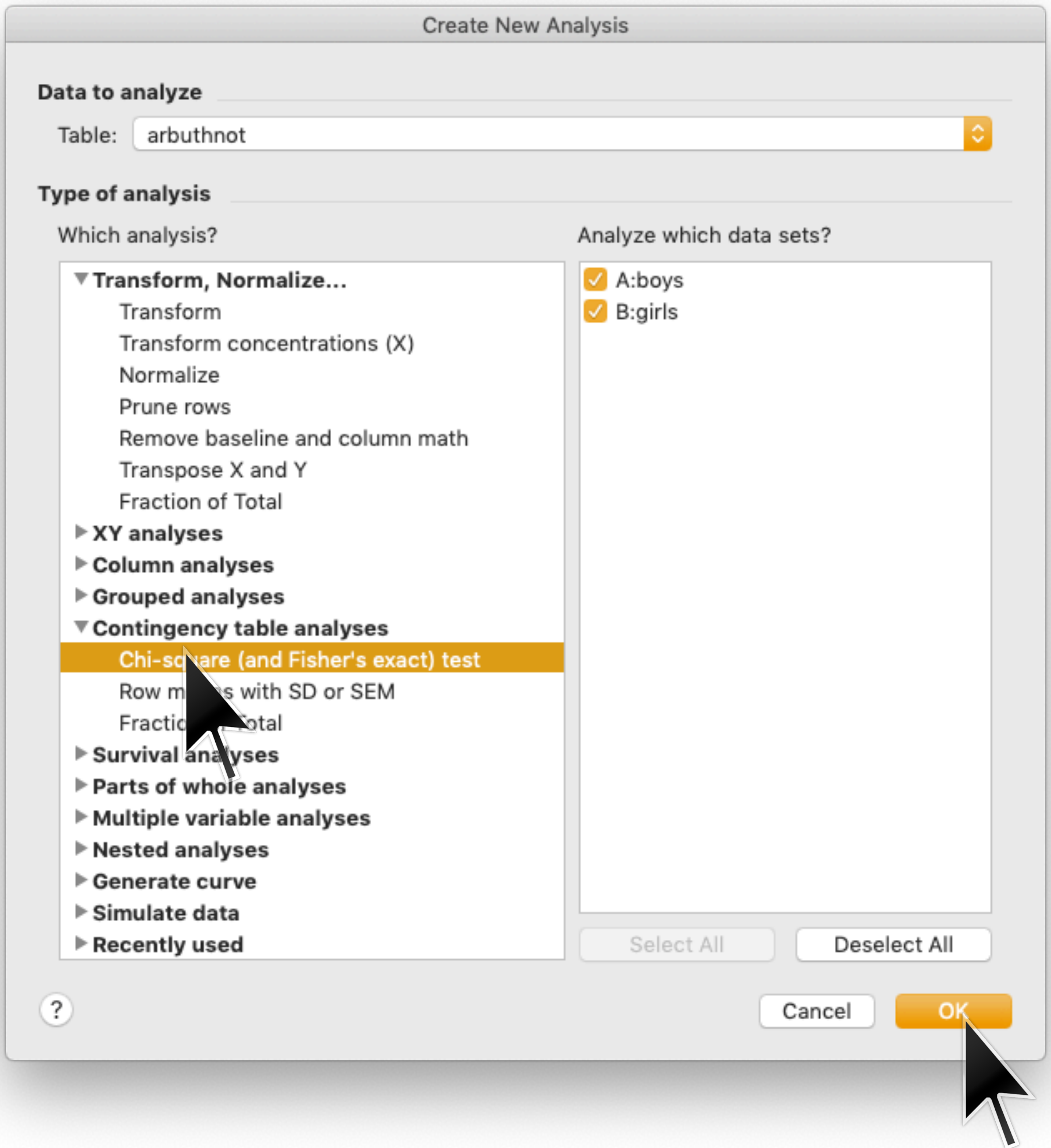
m = number of possible values for A

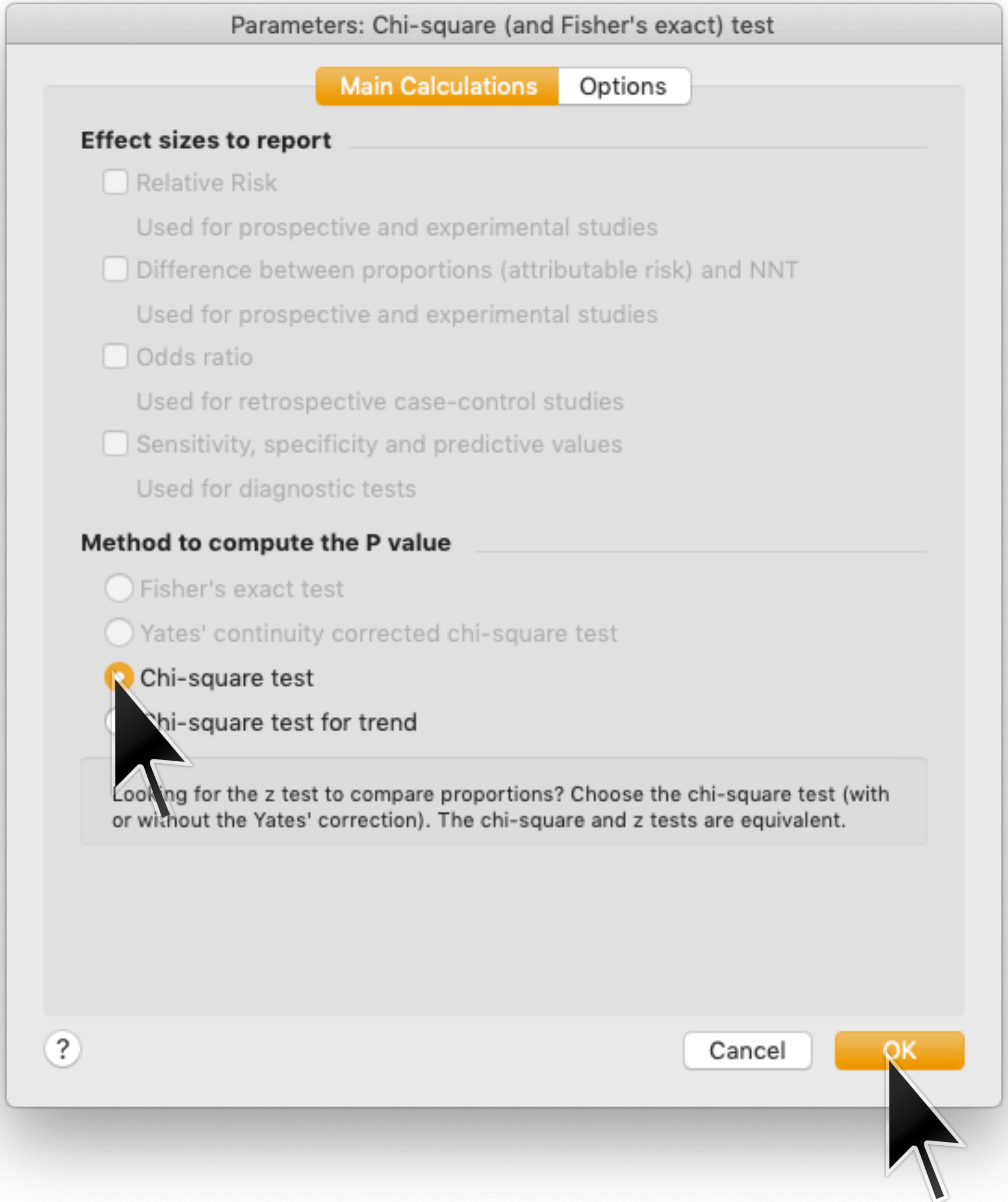
n = number of possible values or B

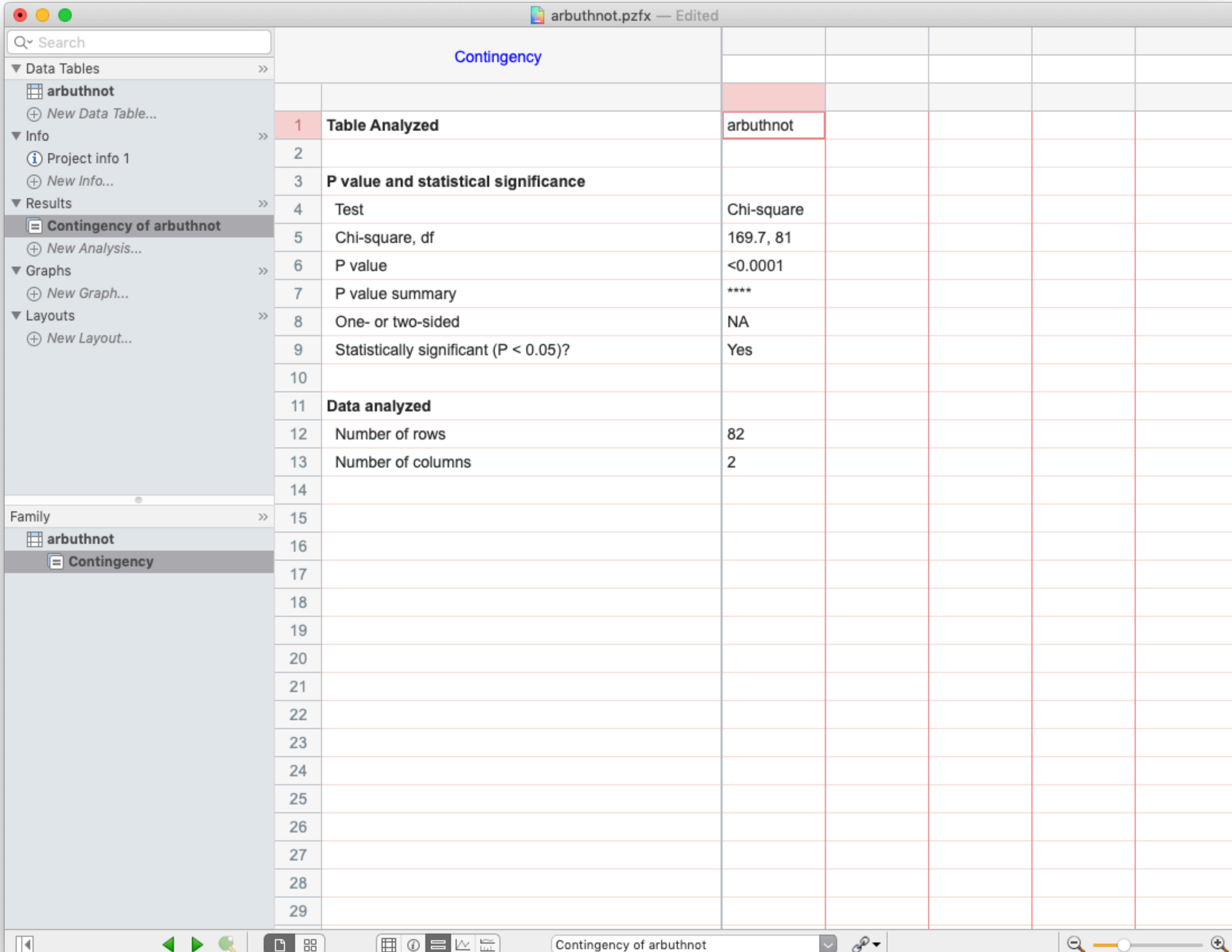

arbuthnot.pzfx
Search
Data Tables
arbuthnot
New Data Table...
Info
Project info 1
New Info...
Results
New Analysis...
G hs
Graph...
L ts
Ne Layout...
Family
arbuthnot
Table format:
Contingency

		Outcome A	Outcome B	Outcome C	Outcome D	Outcome E	Outcome F	Outcome G	Outcome H
		boys	girls	Title	Title	Title	Title	Title	Title
		Y	Y	Y	Y	Y	Y	Y	Y
1	1629	5218	4683						
2	1630	4858	4457						
3	1631	4422	4102						
4	1632	4994	4590						
5	1633	5158	4839						
6	1634	5035	4820						
7	1635	5106	4928						
8	1636	4917	4605						
9	1637	4703	4457						
10	1638	5359	4952						
11	1639	5366	4784						
12	1640	5518	5332						
13	1641	5470	5200						
14	1642	5460	4910						
15	1643	4793	4617						
16	1644	4107	3997						
17	1645	4047	3919						
18	1646	3768	3395						
19	1647	3796	3536						
20	1648	3363	3181						
21	1649	3079	2746						
22	1650	2890	2722						
23	1651	3231	2840						
24	1652	3220	2908						
25	1653	3196	2959						
26	1654	3441	3179						
27	1655	3655	3349						
28	1656	3668	3382						
29	1657	3396	3289						

arbuthnot
Row 6, Column E


Create New Analysis
Data to analyze
Table:
arbuthnot
Type of analysis
Which analysis?
Analyze which data sets?
Transform, Normalize...
Transform
Transform concentrations (X)
Normalize
Prune rows
Remove baseline and column math
Transpose X and Y
Fraction of Total
XY analyses
Column analyses
Grouped analyses
Contingency table analyses
(and Fisher's exact) test
with SD or SEM
Parts of whole analyses
Multiple variable analyses
Nested analyses
Generate curve
Simulate data
Recently used
A:boys
B:girls
Select All
Deselect All
?
Cancel
OK


Parameters: Chi-square (and Fisher's exact) test
Main Calculations
Options
Effect sizes to report
Relative Risk
Used for prospective and experimental studies
Difference between proportions (attributable risk) and NNT
Used for prospective and experimental studies
Odds ratio
Used for retrospective case-control studies
Sensitivity, specificity and predictive values
Used for diagnostic tests
Method to compute the P value
Fisher's exact test
Yates' continuity corrected chi-square test
Chi-square test
Chi-square test for trend
Looking for the z test to compare proportions? Choose the chi-square test (with or without the Yates' correction). The chi-square and z tests are equivalent.
?
Cancel
OK



	Contingency	
1	**Table Analyzed**	arbuthnot
2		
3	**P value and statistical significance**	
4	Test	Chi-square
5	Chi-square, df	169.7, 81
6	P value	<0.0001
7	P value summary	****
8	One- or two-sided	NA
9	Statistically significant (P < 0.05)?	Yes
10		
11	**Data analyzed**	
12	Number of rows	82
13	Number of columns	2